

On the lepton CP violation in a ν 2HDM with flavor

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In this work we propose an extension of Standard Model in which we consider the model 2HDM type-III plus massive neutrinos and the horizontal flavor symmetry S_3 (ν 2HDM-III $\otimes S_3$). In the above framework and with the explicit breaking of flavor symmetry S_3 , the Yukawa matrices in the flavor adapted basis are represented by means of a matrix with two texture zeros. Also, the active neutrinos are considered as Majorana particles and their masses are generated through type I seesaw mechanism. The unitary matrices that diagonalize to the mass matrices, as well as the flavor mixing matrices, are expressed in terms of fermion mass ratios. Consequently, the entries of the Yukawa matrices in the mass basis naturally acquires the form of the so-called *Cheng-Sher ansatz*. For the leptonic sector of ν 2HDM-III $\otimes S_3$, we compare the theoretical expressions of the flavor mixing angles with the current experimental data on masses and flavor mixing of leptons, through a χ^2 likelihood test. The results obtained in this χ^2 analysis are in very good agreement with the current experimental data. We also obtained an allowed value ranges for the “Dirac-like” phase factor, as well as for the two Majorana phase factors. We also study the phenomenological implications of these numerical values of the CP-violation phases on the neutrinoless double beta decay, as well as for long-baseline neutrino oscillation experiments such as T2K, NO ν A, and DUNE.

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I. INTRODUCTION

In concordance with the recent work focus on neutrino physics [1, 2], the absolute neutrino mass scale, if neutrinos are Dirac or Majorana fermions, and the possible CP violation sources in leptons are unsolved questions. Nowadays, responses to these questions are being searched in the framework of experimental results concerning KamLAND reactor neutrinos [3, 4], in each one of the current high-statistics Short Base-Line (SBL) reactor neutrino experiments RENO [5, 6], Double Chooz [7] and Daya Bay [8]. Also, one of the most interesting effects in the neutrino oscillation in matter is that these periodic transformations of neutrinos from one flavor to another, can induce a fake CP violating effect. Therefore, the Long Base-Line (LBL) neutrino oscillation experiments are a good candidate for determining the “Dirac-like” CP violation phase as well as resolving the mass hierarchy [9]. The recent measures reported by T2K [10], Nu ν A [11] and Super-Kamiokande experiments [12] suggest a nearly maximal CP violation. In these experiments the “Dirac-like” CP phase takes the value $\delta_{CP} \simeq 3\pi/2$, with a statistical significance below 3σ level. Moreover, the data obtained in the global fits of neutrino oscillations agrees with a nonzero phase δ_{CP} , whereby the previous hint is confirmed [2, 13–15].

The flavor and mass generation are two concepts strongly intertwined. In order to know the flavor dynamic in models beyond the Standard Model (SM), we need to understand the mechanism of flavor and mass generation arising in the standard theory. In this one, the Yukawa matrices are of great interest because its own values define the fermion masses. Moreover, for multi-Higgs models the Flavor Changing Neutral Current (FCNC) arise from the not simultaneous diagonalization of the mass and Yukawa matrices. In particular, models like the Two Higgs Doublet Model type III (2HDM-III) in which the two Higgs doublets are coupled to all fermions, allow the presence of FCNC at tree level mediated by Higgs [16, 17]. The 2HDM predicts three neutral states $H_{1,2,3}^0$ and a pair of charged states denoted as $H_{1,2}^\pm$ [20]. The Higgs-fermions couplings ($H^0 \bar{f} f$) in the 2HDM are given as [16]:

$$\mathcal{L}_Y = \bar{f}_i (S_{ij} + \gamma^5 P_{ij}) f_j H_a^0, \quad (a = 1, 2, 3).$$

In 2HDM-III the FCNC’s are kept under control by imposing some zero texture Yukawa matrices that reproduce the observed fermion masses and mixing angles [18]. Using texture shapes allow a direct relation of the Yukawa matrix entries mixing with the parameters used to compute the decay widths and cross section without losing the terms proportional to the light fermion masses. Specifically, considering a zero texture Yukawa matrix one obtains the *Cheng-Sher ansatz* for flavor mix couplings, widely used in literature, where flavored couplings are considered proportional to the involved fermion masses [19].

The matter content in the 2HDM is divided in quark and leptonic sectors. In turn these sectors are subdivided in two sectors, the quark-up and -down for quark sector, and charged leptons and neutrinos for the leptonic sector. The fermions in each one of these subsectors are analogous each other, because they have completely identical couplings to all gauge bosons, although their mass values are not the same. Therefore, before of the spontaneous symmetry breaking (SSB), the Yukawa Lagrangian in the above subsectors is invariant under permutations of flavor indices. In other words, each one of these subsectors is invariant under the action of a S_3 symmetry group. This group has only three irreducible representations that correspond to two singlets and a doublet [21, 22].

On the other hand, from the experimental data, the mass spectrum for Dirac fermions obey the follow strong

hierarchy [23]:

$$\begin{array}{lll}
\hat{m}_e \sim 10^{-6}, & \hat{m}_\mu \sim 10^{-3}, & \hat{m}_\tau \sim 1, \\
\hat{m}_u \sim 10^{-5}, & \hat{m}_c \sim 10^{-3}, & \hat{m}_t \sim 1, \\
\underbrace{\hat{m}_d \sim 10^{-3}, \hat{m}_s \sim 10^{-2}}_{\mathbf{2}}, & & \underbrace{\hat{m}_b \sim 1}_{\mathbf{1}}.
\end{array} \tag{1}$$

In the above expression $\hat{m}_l = m_l/m_\tau$ ($l = e, \mu, \tau$) for charged leptons, $\hat{m}_U = m_U/m_t$ ($U = u, c, t$) for u -type quarks, while $\hat{m}_D = m_D/m_b$ ($D = d, s, b$) for d -type quarks. The behaviour of these mass ratios in terms of irreducible representations of some symmetry group, can be interpreted as follow; the two lighter particles are associated with a doublet representation $\mathbf{2}$, whereas and the heaviest particle is assigned a singlet representation $\mathbf{1}$. The smallest non-abelian group with irreducible representations of singlet and doublet is the group of permutations of three objects. Hence, we expect the hierarchical nature of the Dirac fermion mass matrices to have its origin in the representation structure $\mathbf{3} = \mathbf{2} \oplus \mathbf{1}$ of S_3 . In the theoretical framework of SM, as well as for 2HDM, the neutrinos are massless particles, fact that is in disagree with the results obtained in the neutrino oscillation experiments.

Therefore, in this work we will study the flavor dynamics through Yukawa matrices in the specific scenario of 2HDM-III plus massive neutrinos and an horizontal flavor symmetry S_3 (ν 2HDM-III $\otimes S_3$). In this context, the active neutrinos are considered as Majorana particles and their masses are generated through type I seesaw mechanism, while under the action of S_3 symmetry grupo the right-handed neutrinos as well as the two Higgs fields transforms as singlets. After the explicit breaking of flavor symmetry according to the chain $S_{3L}^j \otimes S_{3R}^j \supset S_3^{\text{diag}} \supset S_2^{\text{diag}}$, all Yukawa matrices in the flavor adapted basis are represented by means of a matrix with two texture zeros. Therefore, all fermionic mass matrices in the model have the same generic form with two texture zeros.

The difference between 2HDM and ν 2HDM $\otimes S_3$ models lies in the Yukawa structure, in the symmetries of the Higgs sector and in the possible appearance of new sources of CP violation. This CP violation can arise from the same phase appearing in the Cabibbo-Kobayashi matrix, as in the SM, or some extra phase which arises in the Yukawa field or in the Higgs potential, either explicitly or spontaneously. The Higgs potential preserves CP symmetry, whereby CP violation come from the Yukawa matrices.

In order to validate our hypothesis where the S_3 horizontal flavor symmetry is explicitly breaking, hence all fermion mass matrices are represented through a matrix with two texture zeros, we make a likelihood test where the χ^2 function is defined in terms of leptonic flavor mixing angles. After, we shall investigate the phenomenological implications of these results on the neutrinoless double beta decay and the CP violation in neutrino oscillations in matter.

The plan of this work is a follows. In section II we present the Yukawa lagrangian in the ν 2HDM-III $\otimes S_3$, the form of the Dirac and Majorana fermion mass matrices in function of its eigenvalues. In this way, we derive explicit and analytical expressions for the leptonic flavor mixing angles and Higgs-fermion-fermion couplings. In section III we present a detailed likelihood test where the χ^2 function is defined in terms of leptonic mixing angles. In section IV we explore the phenomenological implications of the numerical values obtained for the CP violating phase factors, on the neutrinoless double beta decay and the neutrino oscillations in matter. Finally, in the section V we present the conclusions and remarks of the present work.

II. THE YUKAWA LAGRANGIAN IN THE $\nu 2\text{HDM} \otimes S_3$

In the matter content of the SM with 2HDM-III there are no right-handed neutrinos, consequently a neutrino mass term is forbidden. In order to include a Majorana neutrino mass term to the 2HDM-III, we need to increase its matter content. For that, we consider six neutrino fields: three left-handed $\nu_L = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})^\top$ and three right-handed $N_R = (N_{1R}, N_{2R}, N_{3R})^\top$. The right-handed neutrinos must be uncharged under both the weak and electromagnetic interactions, which means that this kind of neutrinos are singlets under $G_{SM} = SU(2)_L \otimes U(1)_Y$. So, only the left-handed neutrinos take part in the electroweak interaction. In this theoretical framework named $\nu 2\text{HDM}$ and in the weak eigenstate basis, the Yukawa interaction Lagrangian for Dirac fermions is given by [24, 25]

$$\mathcal{L}_Y = \sum_{k=1}^2 \left(\mathbf{Y}_k^u \bar{Q} \tilde{\Phi}_k u_R + \mathbf{Y}_k^d \bar{Q} \Phi_k d_R + \mathbf{Y}_k^{\nu D} \bar{L} \tilde{\Phi}_k N_R + \mathbf{Y}_k^l \bar{L} \Phi_k l_R \right) + \text{H. c.}, \quad (2)$$

where $Q = (u, d)_L^\top$ and $L = (\nu_l, l)_L^\top$ are the left-handed doublets of $SU(2)_L$; u_R , d_R and l_R are the right-handed singlets of weak gauge group. In this expression the indices l , u and d represent the charged leptons, and u - and d -type quarks, respectively. The $\Phi_k = (\phi_k^+, \phi_k^0)^\top$ denotes the Higgs $SU(2)_L$ doublet field, with $\tilde{\Phi}_k = i\sigma_2 \Phi_k^*$. Finally, the \mathbf{Y}_j^k ($j = u, d, l, \nu_D$) are the Yukawa matrices (j denotes Dirac fermions) [24]. In general, after the SSB, in $\nu 2\text{HDM}$ the Dirac fermion mass matrix in the weak eigenstate basis can be written as [24, 25]

$$\mathbf{M}_k = \frac{1}{\sqrt{2}} \left(v_1 \mathbf{Y}_1^j + v_2 \mathbf{Y}_2^j \right), \quad (3)$$

where v_1 and v_2 are the vacuum expectation values (vev's) of Φ_1 and Φ_2 , respectively.

On the other hand, in $\nu 2\text{HDM}$ we consider that active neutrinos acquire their small mass value through the type I seesaw mechanism. Hence, we consider the hybrid mass term,

$$\mathcal{L}^{M+D} = -\frac{1}{2} \bar{\eta}_L \mathbf{M}^{M+D} (\eta_L)^c + \text{H. c.}, \quad (4)$$

which involves the Dirac and Majorana neutrino mass terms, where $\eta = (\nu_L, (N_R)^c)^\top$ and

$$\mathbf{M}^{M+D} = \begin{pmatrix} \mathbf{0} & \mathbf{M}_D \\ \mathbf{M}_D^\top & \mathbf{M}_R \end{pmatrix}. \quad (5)$$

Here, \mathbf{M}_D and \mathbf{M}_R are the Dirac and right-handed neutrino mass matrix, respectively. In the special limit $\mathbf{M}_R \gg \mathbf{M}_D$, the mass matrix of active neutrinos is given by the type-I seesaw mechanism whose expression is

$$\mathbf{M}_\nu = \mathbf{M}_D \mathbf{M}_R^{-1} \mathbf{M}_D^\top. \quad (6)$$

If the fermion mass matrices have not any element equal to zero, on one hand, the mass matrix of active neutrinos has twelve free parameters, since \mathbf{M}_ν is a complex symmetric matrix because this matrix come from a Majorana mass term. On the other hand, the Dirac fermion mass matrices do not have any special feature, *i.e.*, these matrices are not Hermitian nor symmetric, this is mainly due to the fact that the Yukawa matrices are represented through of 3×3 complex matrix. Hence, for Dirac mass matrices we have eighteen free parameters.

A. Mass matrices from the S_3 flavor symmetry

In order to reduce the free parameters in the fermion mass matrices of the model, we will consider that Yukawa matrices are represented by means of Hermitian matrix, which obey an horizontal symmetry that correlate the particle

flavor indices. Then, as other authors have done in the SM [26–28] (and references in there), we propose that after the SSB, and for three generations or families of quarks and leptons, the Yukawa Lagrangian in $\nu 2\text{HDM}$ has the horizontal flavor symmetry $S_{3L}^j \otimes S_{3R}^j$ ($j = u, d, l, \nu_D$). The two Higgs bosons in $\nu 2\text{HDM}$ are accommodated in a singlet representation of S_3 . This implies that these two $SU(2)_L$ doublets have no flavor, whereby these fields are treated as scalars with respect to the S_3 transformations. Also, under the action of flavor symmetry $S_{3L}^j \otimes S_{3R}^j$, the left- and right-handed spinors transform as [27]:

$$\psi_{jL}^s = \mathbf{g}_i^j \psi_{jL} \quad \text{and} \quad \psi_{jR}^s = \tilde{\mathbf{g}}_i^j \psi_{jR}, \quad (7)$$

where $\mathbf{g}_i^j \in S_{3L}^j$ and $\tilde{\mathbf{g}}_i^j \in S_{3R}^j$, while the superscript “s” means that fields are in the flavor symmetry adapted basis. The elements of $S_{3L}^j \otimes S_{3R}^j$ flavor group are the pairs $(\mathbf{g}_i^j, \tilde{\mathbf{g}}_i^j)$. However, as the mass terms and charged currents in the Lagrangian, $\mathcal{L}_m \sim \bar{\psi}_{jL} \mathbf{M}_j \psi_{jR}$ and $\mathcal{J}_\mu \sim \bar{\psi}_{jL} \gamma_\mu \psi_{jL} W^\mu$, as well as the fermion mass matrices must be invariant under the action of the flavor group transformations, the elements of the flavor group $(\mathbf{g}_i^j, \tilde{\mathbf{g}}_i^j)$ must satisfy the relation $\mathbf{g} \equiv \mathbf{g}_i = \tilde{\mathbf{g}}_j$. This condition implies that the flavor group is reduced according the chain: $S_{3L}^k \otimes S_{3R}^k \supset S_3^{\text{diag}}$ [27]. This chain of flavor symmetry breaking means that all fermion fields must be transformed with the same flavor group and the same element group (\mathbf{g}, \mathbf{g}) . In the previous expression $\mathbf{g} \in S_{3L}$ and the set of pairs (\mathbf{g}, \mathbf{g}) is called S_3^{diag} [27]. Also, from the invariance of the Lagrangian \mathcal{L}_m under the action of group S_3^{diag} , we obtain that the mass matrices commute with all elements of the flavor group.

In the weak eigenstate basis the two Yukawa matrices in Eq. (3) are represented by means of a matrix with the exact S_3^{diag} symmetry. Hence, the Yukawa matrices are expressed as

$$\mathbf{Y}_k^j = \alpha_k^j \mathbf{P}_1, \quad (8)$$

where α_k^j is a real constant. The explicit form of \mathbf{P}_1 is given in Eq. (A1), and corresponds to the projector associated with the symmetric singlet representation. Hence, the mass matrix for Dirac fermion is $\mathbf{M}_j = m_{j3} \mathbf{P}_1$, where $m_{j3} = \frac{1}{\sqrt{2}} (v_1 \alpha_1^j + v_2 \alpha_2^j)$. In the flavor adapted basis, the mass matrices are [29]

$$\mathbf{M}_j^S = \mathbf{U}_S^\dagger \mathbf{M}_j \mathbf{U}_S = m_{j3} \mathbf{U}_S^\dagger \mathbf{P}_1 \mathbf{U}_S = \text{diag}(0, 0, m_{j3}), \quad (9)$$

where

$$\mathbf{U}_S = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 1 & \sqrt{2} \\ -\sqrt{3} & 1 & \sqrt{2} \\ 0 & -2 & \sqrt{2} \end{pmatrix}. \quad (10)$$

The interpretation of Eq. (9) is that under an exact S_3^{diag} symmetry, the mass spectrums for Dirac fermions consist of a massive particle and two massless particles [30]. The massive particle in each one of the mass spectra corresponds to the third heaviest fermion in each generation. However, this result is in disagreement with the experimental data on quark and lepton masses [23].

So, with the aim of generate a nonzero mass for all fermions in the model, we will break the flavor symmetry in an explicit way. Respectively, the first two fermion families and third one are assigned to the doublet and singlet representation of S_3^{diag} . The mass to second fermion family is generated when the S_3^{diag} flavor symmetry is explicitly broken to S_2^{diag} by adding the following term to \mathbf{Y}_k^j , Eq. (8):

$$\mathbf{Y}_k^{j2} = \beta_k^j \mathbf{T}_{z1}^+ + \gamma_k^j \mathbf{T}_{z2}^+, \quad (11)$$

where β_k^j and γ_k^j are real constant parameters. The explicit form of \mathbf{T}_{z1}^+ and \mathbf{T}_{z2}^+ is given in Eq. (A10). The \mathbf{Y}_k^{j2} matrix mixes the symmetric component of the doublet with the singlet. Finally, the first fermion family's mass is generated by adding the term

$$\mathbf{Y}_k^{j1} = \epsilon_k^j \mathbf{T}_x^+ + \rho_k^j \mathbf{T}_x^-, \quad (12)$$

where ϵ_k^j and ρ_k^j are real constant parameters. The explicit form of \mathbf{T}_x^+ and \mathbf{T}_x^- is given in Eq. (A6). The \mathbf{Y}_k^{j1} matrix mixes the components of the doublet representation with each other. In the weak eigenstates basis and under the explicit breaking of flavor symmetry according to the chain $S_{3L}^j \otimes S_{3R}^j \supset S_3^{\text{diag}} \supset S_2^{\text{diag}}$, the Yukawa matrices are the sum of the three expressions given in eqs. (8), (11), and (12). Then,

$$\begin{aligned} \mathbf{Y}_k^{w,j} &= \mathbf{Y}_k^j + \mathbf{Y}_k^{j2} + \mathbf{Y}_k^{j1} = \alpha_k^j \mathbf{P}_1 + \beta_k^j \mathbf{T}_{z1}^+ + \gamma_k^j \mathbf{T}_{z2}^+ + \epsilon_k^j \mathbf{T}_x^+ + \rho_k^j \mathbf{T}_x^-, \\ \mathbf{Y}_k^{w,j} &= \begin{pmatrix} e_k^{w,j} & a_k^{w,j} & f_k^{w,j} \\ a_k^{w,j*} & b_k^{w,j} & c_k^{w,j} \\ f_k^{w,j*} & c_k^{w,j*} & d_k^{w,j} \end{pmatrix}, \end{aligned} \quad (13)$$

where

$$\begin{aligned} a_k^{w,j} &= \frac{\alpha_k^j + 3(\beta_k^j + i\rho_k^j)}{3}, \quad b_k^{w,j} = \frac{\alpha_k^j + 3(\beta_k^j + \epsilon_k^j)}{3}, \quad c_k^{w,j} = \frac{\alpha_k^j + 3(\gamma_k^j - \epsilon_k^j + i\rho_k^j)}{3}, \\ d_k^{w,j} &= \frac{\alpha_k^j - 6\beta_k^j}{3}, \quad e_k^{w,j} = \frac{\alpha_k^j + 3(\beta_k^j - \epsilon_k^j)}{3}, \quad f_k^{w,j} = \frac{\alpha_k^j + 3(\gamma_k^j + \epsilon_k^j - i\rho_k^j)}{3}. \end{aligned} \quad (14)$$

In this same basis, the Dirac fermion mass matrices are

$$\mathbf{M}_j = \sum_{k=1}^2 \frac{v_k}{\sqrt{2}} \mathbf{Y}_k^{w,j}. \quad (15)$$

Hence, with help of the previous expression and Eq. (13), it is easy to conclude that the matrices \mathbf{M}_j are Hermitian matrices without any of their elements being equal to zero. However, in the flavor adapted basis the mass matrices are

$$\begin{aligned} \mathbf{M}_j^S &= \mathbf{U}_S^\dagger \mathbf{M}_j \mathbf{U}_S = \sum_{k=1}^2 \frac{v_k}{\sqrt{2}} \mathbf{U}_S^\dagger \mathbf{Y}_k^{w,j} \mathbf{U}_S, \\ \mathbf{M}_j^S &= \mathbf{P}_j \begin{pmatrix} 0 & |A_j| & 0 \\ |A_j| & B_j & C_j \\ 0 & C_j & D_j \end{pmatrix} \mathbf{P}_j^\dagger = \frac{v \cos \beta}{\sqrt{2}} \times \left[\begin{pmatrix} 0 & A_1^j & 0 \\ A_1^{j*} & B_1^j & C_1^j \\ 0 & C_1^{j*} & D_1^j \end{pmatrix} + \tan \beta \begin{pmatrix} 0 & A_2^j & 0 \\ A_2^{j*} & B_2^j & C_2^j \\ 0 & C_2^{j*} & D_2^j \end{pmatrix} \right], \end{aligned} \quad (16)$$

where $\mathbf{P}_j = \text{diag}(1, e^{-i\phi_j}, e^{-i\phi_j})$, $\phi_j = \arg\{A_j\}$, and

$$A_k^j = -\sqrt{3}(\epsilon_k^j - i\rho_k^j), \quad B_k^j = -\frac{2}{3}(\beta_k^j + 2\gamma_k^j), \quad C_k^j = \frac{\sqrt{2}}{3}(4\beta_k^j - \gamma_k^j), \quad D_k^j = \alpha_k^j + \frac{2}{3}(\beta_k^j + 2\gamma_k^j), \quad (k=1,2). \quad (17)$$

Finally, $\tan \beta = \frac{v_2}{v_1}$ and $v^2 = v_1^2 + v_2^2 = (246.22 \text{ GeV})^2$.

In this work we consider that active neutrinos acquire their mass through the type I seesaw mechanism (6), where the Dirac fermion mass matrix is given by Eq. (16), while the right-handed neutrino mass matrix is the real symmetric matrix $M_R = \text{diag}(A_R, A_R, D_R) \mathbf{D}^3(A_1)$; the matrix $\mathbf{D}^3(A_1)$ is given in Eq. (A1). Hence, in the flavor adapted basis

the active neutrinos mass matrix is

$$\mathbf{M}_\nu = \mathbf{P}_\nu^\dagger \begin{pmatrix} 0 & a_\nu & 0 \\ a_\nu & |b_\nu| & |c_\nu| \\ 0 & |c_\nu| & d_\nu \end{pmatrix} \mathbf{P}_\nu^\dagger, \quad (18)$$

where $\mathbf{P}_\nu = e^{i\phi_\nu} \text{diag}(1, e^{-i2\phi_\nu}, e^{-i\phi_\nu})$, $\phi_\nu = \arg\{C_\nu\}$, with $\arg\{C_\nu\} = 2\arg\{B_\nu\}$,

$$A_\nu = \frac{|A_{\nu D}|^2}{A_R}, \quad B_\nu = \frac{C_{\nu D}^2}{D_R} + \frac{2B_{\nu D}A_{\nu D}^*}{A_R}, \quad C_\nu = \frac{C_{\nu D}D_{\nu D}}{D_R} + \frac{C_{\nu D}A_{\nu D}^*}{A_R}, \quad \text{and} \quad D_\nu = \frac{D_{\nu D}^2}{D_R}. \quad (19)$$

B. The mass and mixing matrices as function of fermion masses

For a normal [inverted] hierarchy¹ in the mass spectrum the real symmetric matrices in eqs. (16) and (18), which are associated with the fermion mass matrices, can be reconstructed through the orthogonal transformation²

$$\bar{\mathbf{M}}_f^{n[i]} = \mathbf{O}_f^{n[i]} \mathbf{\Delta}_f^{n[i]} \left(\mathbf{O}_f^{n[i]} \right)^\top, \quad (20)$$

where $\mathbf{\Delta}_f^{n[i]} = \text{diag}(m_{f1[3]}, m_{f2[1]}, m_{f3[2]})$. Here, the m_f 's are the eigenvalues of fermion mass matrices, *i.e.*, the particle masses. From algebraic invariants of expression in Eq. (20) we have

$$\begin{aligned} a_f^{n[i]} &\equiv \frac{(\mathbf{M}_f)_{12}}{m_{f3[2]}} = \sqrt{\frac{\hat{m}_{f1[3]}\hat{m}_{f2[1]}}{1-\delta_f}}, & b_f^{n[i]} &\equiv \frac{(\mathbf{M}_f)_{22}}{m_{f3[2]}} = \hat{m}_{f1[3]} - \hat{m}_{f2[1]} + \delta_f, \\ c_f^{n[i]} &\equiv \frac{(\mathbf{M}_f)_{23}}{m_{f3[2]}} = \sqrt{\frac{\delta_f}{1-\delta_f}\xi_{f1[3]}\xi_{f2[1]}}, & d_f^{n[i]} &\equiv \frac{(\mathbf{M}_f)_{33}}{m_{f3[2]}} = 1 - \delta_f, \end{aligned} \quad (21)$$

where $\xi_{f1[3]} = 1 - \hat{m}_{f1[3]} - \delta_f$ and $\xi_{f2[1]} = 1 + \hat{m}_{f2[1]} - \delta_f$, with $\hat{m}_{f1[3]} = m_{f1[3]}/m_{f3[2]}$, $\hat{m}_{f2[1]} = |m_{f2[1]}|/m_{f3[2]}$, with $f = u, d, l, \nu$. For $m_{f2[1]} = -|m_{f2[1]}|$, we have that the free parameter δ_f must satisfy the relation $1 - \hat{m}_{f1[3]} > \delta_f > 0$. The real orthogonal matrix given in Eq. (20) in terms of fermion masses has the shape [25, 28]

$$\mathbf{O}_f^{n[i]} = \begin{pmatrix} \sqrt{\frac{\hat{m}_{f2[1]}\xi_{f1[3]}}{\mathcal{D}_{f1[3]}}} & -\sqrt{\frac{\hat{m}_{f1[3]}\xi_{f2[1]}}{\mathcal{D}_{f2[1]}}} & \sqrt{\frac{\hat{m}_{f1[3]}\hat{m}_{f2[1]}\delta_f}{\mathcal{D}_{f3[2]}}} \\ \sqrt{\frac{\hat{m}_{f1[3]}(1-\delta_f)\xi_{f1[3]}}{\mathcal{D}_{f1[3]}}} & \sqrt{\frac{\hat{m}_{f2[1]}(1-\delta_f)\xi_{f2[1]}}{\mathcal{D}_{f2[1]}}} & \sqrt{\frac{\delta_f(1-\delta_f)}{\mathcal{D}_{f3[2]}}} \\ -\sqrt{\frac{\hat{m}_{f1[3]}\delta_f\xi_{f2[1]}}{\mathcal{D}_{f1[3]}}} & -\sqrt{\frac{\hat{m}_{f2[1]}\delta_f\xi_{f1[3]}}{\mathcal{D}_{f2[1]}}} & \sqrt{\frac{\xi_{f1[3]}\xi_{f2[1]}}{\mathcal{D}_{f3[2]}}} \end{pmatrix}. \quad (22)$$

In this orthogonal matrix we have

$$\begin{aligned} \mathcal{D}_{f1[3]} &= (1 - \delta_f) (\hat{m}_{f1[3]} + \hat{m}_{f2[1]}) (1 - \hat{m}_{f1[3]}), \\ \mathcal{D}_{f2[1]} &= (1 - \delta_f) (\hat{m}_{f1[3]} + \hat{m}_{f2[1]}) (1 + \hat{m}_{f2[1]}), \\ \mathcal{D}_{f3[2]} &= (1 - \delta_f) (1 - \hat{m}_{f1[3]}) (1 + \hat{m}_{f2[1]}). \end{aligned} \quad (23)$$

In the theoretical framework of $\nu 2\text{HDM} \otimes S_3$, the unitary matrices that diagonalize the mass matrices of charged leptons and active neutrinos are defined as

$$\mathbf{U}_\ell^{n[i]} = \mathbf{U}_S \mathbf{P}_\ell \mathbf{O}_\ell^{n[i]} \quad (\ell = l, \nu). \quad (24)$$

¹ The inverted hierarchy is only valid for neutrinos.

² The superscript $n[i]$ denote the normal [inverted] hierarchy in the neutrino mass spectrum.

The matrix \mathbf{U}_S is given in Eq. (10), and the diagonal phase matrices are given in eqs. (16) and (18). Finally, the real orthogonal matrix $\mathbf{O}_\ell^{n[i]}$ is given in Eq. (22). From the previous unitary matrix, in the mass states basis the Dirac fermion mass matrices take the shape

$$\Delta_j = \mathbf{U}_j^\dagger \mathbf{M}_j \mathbf{U}_j = \sum_{k=1}^2 \frac{v_k}{\sqrt{2}} \mathbf{U}_j^\dagger \mathbf{Y}_k^{w,j} \mathbf{U}_j = \sum_{k=1}^2 \frac{v_k}{\sqrt{2}} \tilde{\mathbf{Y}}_k^j \quad (25)$$

where $\tilde{\mathbf{Y}}_k^j = \mathbf{U}_j^\dagger \mathbf{Y}_k^{w,j} \mathbf{U}_j$ are the Yukawa matrices in the mass basis. Now, with the help of the orthogonal matrix given in Eq. (22) and, as the mass spectrum of Dirac particles only has the normal hierarchy, it is easy to obtain that elements of Yukawa matrices $\tilde{\mathbf{Y}}_k^j$ can be expressed in terms of geometric mean of Dirac fermion masses normalized with respect of electroweak scale, *i.e.*,

$$\left(\tilde{\chi}_k^j \right)_{\mathbf{rs}} = \frac{\sqrt{m_{j\mathbf{r}} m_{j\mathbf{s}}}}{v} \left(\tilde{\chi}_k^j \right)_{\mathbf{rs}} \quad (\mathbf{r}, \mathbf{s} = 1, 2, 3). \quad (26)$$

Here, $\tilde{\chi}_k^j$ are complex symmetric matrices, the explicit form of the entries of these matrices are given in Appendix B. In the literature, this latter relation is called *Cheng-Sher ansatz* [31], which is associated with Higgs-fermion-fermion couplings. Also, the result obtained in Eq. (26) can be extended to any Hermitian mass matrix that may be brought to a two zero texture matrix by means of an unitary transformation.

The lepton flavor mixing matrix is defined as $\mathbf{U}_{\text{PMNS}} = \mathbf{U}_l^\dagger \mathbf{U}_\nu$ [32], hence, in this context the PMNS matrix takes the form

$$\mathbf{U}_{\text{PMNS}} = \mathbf{O}_l^\top \mathbf{P}^{(\nu-l)} \mathbf{O}_\nu^{n[i]}, \quad (27)$$

where $\mathbf{P}^{(\nu-l)} = \text{diag}(1, e^{i\phi_{\ell 1}}, e^{i\phi_{\ell 2}})$ with $\phi_{\ell 1} = \phi_l - 2\phi_\nu$ and $\phi_{\ell 2} = \phi_l - \phi_\nu$. the explicit form of the entries of the PMNS matrix are given in Appendix C. The first conclusion of this form of PMNS matrix is that the \mathbf{U}_S unitary matrix that allows us to pass from the weak basis to the flavor symmetry adapted basis, is unobservable in the lepton flavor mixing matrix.

C. The mixing angle and CP violation phases

In symmetric parametrization of lepton flavor mixing matrix, the relation between mixing angles and the entries of PMNS matrix is [33]

$$\sin^2 \theta_{13} \equiv |(\mathbf{U}_{\text{PMNS}})_{13}|^2, \quad \sin^2 \theta_{12} \equiv \frac{|(\mathbf{U}_{\text{PMNS}})_{12}|^2}{1 - |(\mathbf{U}_{\text{PMNS}})_{13}|^2}, \quad \sin^2 \theta_{23} \equiv \frac{|(\mathbf{U}_{\text{PMNS}})_{23}|^2}{1 - |(\mathbf{U}_{\text{PMNS}})_{13}|^2}. \quad (28)$$

On the one hand, the lepton Jarlskog invariant which appears in conventional neutrino oscillations is defined as:

$$\mathcal{J}_{CP} = \text{Im} \{ (\mathbf{U}_{\text{PMNS}})_{11}^* (\mathbf{U}_{\text{PMNS}})_{23}^* (\mathbf{U}_{\text{PMNS}})_{13} (\mathbf{U}_{\text{PMNS}})_{21} \}, \quad (29)$$

and in the symmetric parametrization it has the form

$$\mathcal{J}_{CP} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta_{CP}, \quad (30)$$

where $\delta_{CP} = \phi_{13} - \phi_{23} - \phi_{12}$. On the other hand, the invariants

$$\mathcal{I}_1 = \text{Im} \{ (\mathbf{U}_{\text{PMNS}})_{12}^2 (\mathbf{U}_{\text{PMNS}})_{11}^{*2} \} \quad \text{and} \quad \mathcal{I}_2 = \text{Im} \{ (\mathbf{U}_{\text{PMNS}})_{13}^2 (\mathbf{U}_{\text{PMNS}})_{11}^{*2} \}, \quad (31)$$

associated with the Majorana phases [34–36] take the expression

$$\mathcal{I}_1 = \frac{1}{4} \sin^2 2\theta_{12} \cos^4 \theta_{13} \sin(-2\phi_{12}) \quad \text{and} \quad \mathcal{I}_2 = \frac{1}{4} \sin^2 2\theta_{13} \cos^2 \theta_{12} \sin(-2\phi_{13}). \quad (32)$$

Then, the phase factors associated with the CP violation can be written as:

$$\sin(\delta_{CP}) = \frac{\mathcal{I}_{CP} (1 - |(\mathbf{U}_{PMNS})_{13}|^2)}{|(\mathbf{U}_{PMNS})_{11}| |(\mathbf{U}_{PMNS})_{12}| |(\mathbf{U}_{PMNS})_{13}| |(\mathbf{U}_{PMNS})_{23}| |(\mathbf{U}_{PMNS})_{33}|}, \quad (33)$$

$$\sin(-2\phi_{12}) = \frac{\mathcal{I}_1}{|(\mathbf{U}_{PMNS})_{11}|^2 |(\mathbf{U}_{PMNS})_{12}|^2}, \quad \sin(-2\phi_{13}) = \frac{\mathcal{I}_2}{|(\mathbf{U}_{PMNS})_{11}|^2 |(\mathbf{U}_{PMNS})_{13}|^2}.$$

The equivalence between the PDG and symmetric parameterization may be expressed as $\mathbf{U}_{PDG} = \mathbf{K} \mathbf{U}_{Sym}$, where

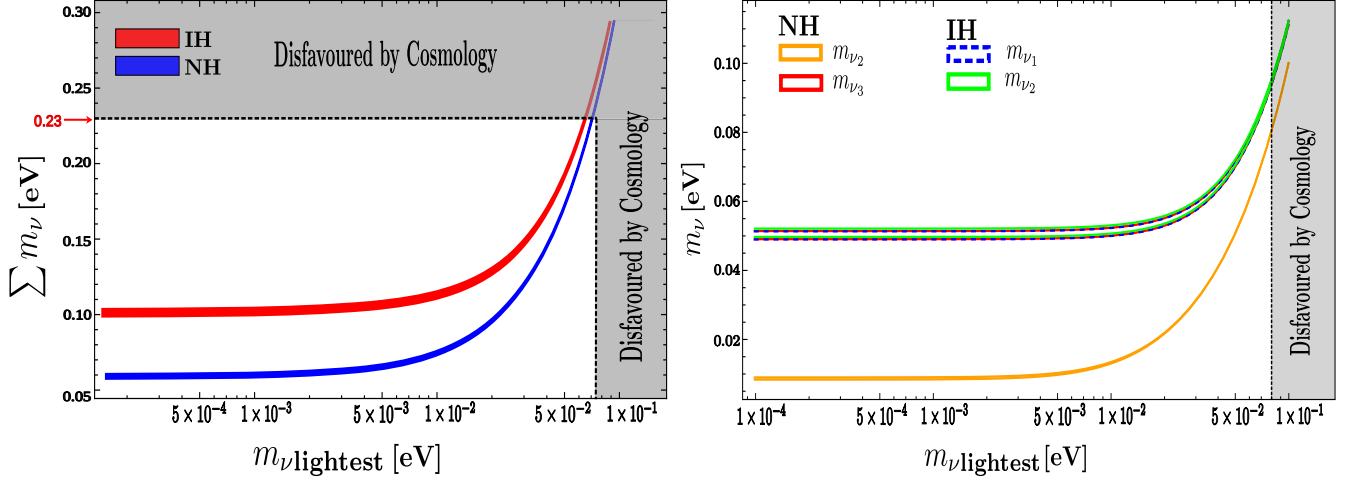


FIG. 1: In the left-panel we show the sum of neutrino masses. The right-panel shows the neutrino masses. The neutrino oscillation parameters Δm_{ij}^2 are taken within the currently allowed 3σ range [15]. The upper bound on the mass of the lightest neutrino is obtained from the Planck result where $\sum m_{\nu_i} < 0.23$ eV at 95% level [37].

$$\mathbf{K} = \text{diag} \left(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}} \right) \text{ with } \delta_{CP} = \phi_{13} - \phi_{23} - \phi_{12}, \alpha_{21} = -2\phi_{12} \text{ and } \alpha_{31} = -2(\phi_{12} + \phi_{23}).$$

III. NUMERICAL ANALYSIS

In the three flavor neutrino scheme there are six independent parameters which rule the behavior of neutrino oscillation phenomena: flavor mixing angles, the “Dirac-like” CP-violating phase and squared-mass splitting. The latest neutrino oscillation parameter is defined as $\Delta m_{ij}^2 \equiv m_{\nu_i}^2 - m_{\nu_j}^2$. In agreement with the results obtained in the global fit reported in Ref. [15]. These neutrino oscillation parameters have the following numerical values (at $\text{BFP} \pm 1\sigma$ and 3σ)³

$$\begin{aligned} \Delta m_{21}^2 \text{ (} 10^{-5} \text{ eV}^2 \text{)} &= 7.50_{-0.17}^{+0.19}, 7.03 - 8.09, \\ \Delta m_{31}^2 \text{ (} 10^{-3} \text{ eV}^2 \text{)} &= 2.524_{-0.040}^{+0.039}, 2.407 - 2.643, \text{ for NH,} \\ \Delta m_{23}^2 \text{ (} 10^{-3} \text{ eV}^2 \text{)} &= 2.514_{-0.041}^{+0.038}, 2.399 - 2.635, \text{ for IH.} \end{aligned} \quad (34)$$

³ Here, NH and IH denote the normal and inverted hierarchy in neutrino mass spectrum, respectively.

	Δm_{ij}^2 at	$\Phi_{\ell 1}$ [°]	$\Phi_{\ell 2}$ [°]	$m_{\nu_{\text{lightest}}}$ [eV]	δ_ℓ	δ_ν	θ_{12}^{th} [°]	θ_{23}^{th} [°]	θ_{13}^{th} [°]	δ_{CP} [°]	ϕ_{12} [°]	ϕ_{13} [°]	χ_{min}^2
NH	3σ	270	195	2.57×10^{-3}	0.20460	0.63519	33.58	41.60	8.47	-68.65	-5.86	14.77	4.63×10^{-4}
	BFP $\pm 1\sigma$	270	195	2.57×10^{-3}	0.22256	0.64507	33.59	41.61	8.46	-70.74	-5.79	14.67	3.13×10^{-4}
	BFP	270	195	2.57×10^{-3}	0.21492	0.64008	33.80	41.63	8.45	-69.85	-5.80	14.73	8.75×10^{-2}
IH	3σ	290	187	2.49×10^{-2}	0.59943	0.01999	33.67	50.08	8.48	-80.90	-5.25	-2.18	2.30×10^{-2}
	BFP $\pm 1\sigma$	290	187	2.49×10^{-2}	0.59888	0.01995	33.74	50.05	8.49	-80.88	-5.25	-2.18	4.56×10^{-2}
	BFP	290	187	2.49×10^{-2}	0.59798	0.01971	33.83	49.99	8.49	-80.83	-5.25	-2.19	1.08×10^{-1}

TABLE I: Numerical values obtained in the BFP for the five parameters, the lepton mixing angles and the phase factors associated with the CP violation. These results were obtained considering to Δm_{ij}^2 at BFP, BFP $\pm\sigma$ and 3σ range [15], and simultaneously $\Phi_{\ell 1}$, $\Phi_{\ell 2}$, δ_ℓ , δ_ν , and $m_{\nu_{1[3]}}$ are free parameters in the χ^2 function.

From the definition of squared-mass splitting Δm_{ij}^2 two of neutrino masses can be written as:

$$m_{\nu_{3[2]}} = \sqrt{m_{\nu_{1[3]}}^2 + \Delta m_{31[23]}^2}, \quad \text{and} \quad m_{\nu_{2[1]}} = \sqrt{m_{\nu_{1[3]}}^2 + \Delta m_{21[31]}^2}. \quad (35)$$

where $m_{\nu_{1[3]}}$ is the lightest neutrino mass⁴. Also, this mass is considered like only free parameter in the above expressions, since the mass-squared Δm_{ij}^2 are determined for experiment.

From the results for cosmological parameters reported by Planck Collaboration, the upper limit on the active neutrino masses sum is $\sum m_{\nu_i} < 0.23$ eV, for a active neutrinos number equal to $N_{eff} = 3.15 \pm 0.23$ [37]. This upper bound is independent of hierachy in the neutrino mass spectrum. So, with all above experimental information and considering the expressions in Eq. (35), we can obtain the following value range for neutrino masses

$$m_{\nu_1} (10^{-2} \text{ eV}) = \begin{cases} [0.00, 7.10] \\ [4.90, 8.25] \end{cases}, \quad m_{\nu_2} (10^{-2} \text{ eV}) = \begin{cases} [0.84, 7.13] \\ [4.97, 8.30] \end{cases}, \quad m_{\nu_3} (10^{-2} \text{ eV}) = \begin{cases} [4.80, 8.75] \\ [0, 6.45] \end{cases}. \quad (36)$$

Here, the oscillation parameters Δm_{ij}^2 are taken within the currently allowed 3σ range [15]. The values in the first and second row correspond to a normal and inverted hierarchy in the neutrino mass spectrum, respectively. For both hierarchies there is the possibility that the lightest neutrino mass could be zero. Namely in this case the lightest neutrino is a massless particle. In the Fig. 1, one shows the behavior of neutrino masses (right-panel) as well as of the neutrino masses sum as function of lightest neutrino mass $m_{\nu_{1[3]}}$ (left-panel).

A. The likelihood test χ^2

In order to validate our hypothesis where the S_3 horizontal flavor symmetry is explicitly breaking according to the chain $S_{3L}^j \otimes S_{3R}^j \supset S_3^{\text{diag}} \supset S_2^{\text{diag}}$, hence all fermion mass matrices are represented through a matrix with two texture zeros, we make a likelihood test where the χ^2 function is defined as:

$$\chi^2 = \sum_{i < j}^3 \frac{\left(\sin^2 \theta_{ij}^{\text{exp}} - \sin^2 \theta_{ij}^{\text{th}} \right)^2}{\sigma_{\theta_{ij}}^2}. \quad (37)$$

⁴ The subscript $i[j]$ denote to normal [inverted] hierarchy in the neutrino masses spectrum.

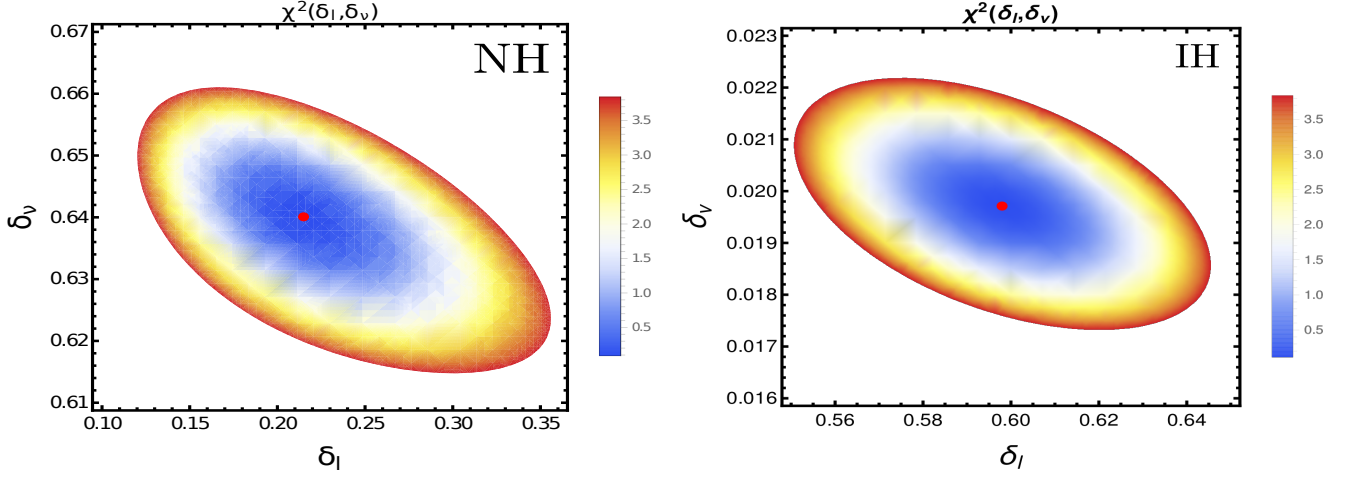


FIG. 2: The allowed regions in the parameter space for δ_l and δ_ν at 3σ C.L., the red point (•) represent to the BFP. The left panel is for normal hierarchy, while the right panel is for the inverted hierarchy. Here, the $m_{\nu_{1[3]}}$, ϕ_{l1} , and ϕ_{l2} parameters are fixing to the values obtained when the Δm_{ij}^2 neutrino oscillation parameters are given at BFP, see Table I.

Here, the “th” superscript is used to denote the lepton mixing angles theoretical expressions, while the terms with superscript “exp” denote to the experimental data with uncertainty $\sigma_{\theta_{ij}}$ for lepton mixing angles. For these latter we consider the following values, at $\text{BFP} \pm 1\sigma$ [15]:

$$\sin^2 \theta_{12}^{\text{exp}} (10^{-1}) = 3.06 \pm 0.12, \quad \sin^2 \theta_{23}^{\text{exp}} (10^{-1}) = \begin{cases} 4.41^{+0.27}_{-0.21} \\ 5.87^{+0.20}_{-0.24} \end{cases}, \quad \sin^2 \theta_{13}^{\text{exp}} (10^{-2}) = \begin{cases} 2.166 \pm 0.0075 \\ 2.179 \pm 0.0076 \end{cases}, \quad (38)$$

the values in the first and second row correspond to a normal and inverted hierarchy in the neutrino mass spectrum, respectively. From expressions in eqs. (22), (27), (28), and (35) it is easy conclude that the χ^2 function depends of five free parameters $\chi^2 = \chi^2(\Phi_{\ell 1}, \Phi_{\ell 2}, \delta_\ell, \delta_\nu, m_{\nu_{1[3]}})$. However, the χ^2 function depends just of three experimental data which correspond to the leptonic flavor mixing angles. Therefore, if simultaneously we consider to $\Phi_{\ell 1}$, $\Phi_{\ell 2}$, δ_ℓ , δ_ν , and $m_{\nu_{1[3]}}$ as free parameters in the likelihood test, we can only determine the values of these parameters in the best fit point (BFP). In accordance with the above, we first seek the BFP by means of a likelihood test where the χ^2 function have to the five free parameters $\Phi_{\ell 1}$, $\Phi_{\ell 2}$, δ_ℓ , δ_ν , and $m_{\nu_{1[3]}}$. For minimize the χ^2 function we done a scanning of parameter space where we considered the following values for the charged lepton masses [23]

$$m_e = 0.5109998928 \pm 0.000000011, \quad m_\mu = 105.6583715 \pm 0.00000035, \quad m_\tau = 1776.82 \pm 0.16, \quad (39)$$

while in Eq. (38) are given the experimental values for leptonic mixing angles.

In the Table I we show the numerical values obtained in the BFP for the five parameters, the lepton mixing angles and the phase factors associated with the CP violation. All these results were obtained considering to Δm_{ij}^2 at BFP, $\text{BFP} \pm \sigma$ and 3σ range, and simultaneously $\Phi_{\ell 1}$, $\Phi_{\ell 2}$, δ_ℓ , δ_ν , and $m_{\nu_{1[3]}}$ are free parameters in the χ^2 function of the likelihood test.

Now, as we know the numerical values of the five free parameters at BFP, we performance a new χ^2 analysis where we fix to the $m_{\nu_{1[3]}}$, $\Phi_{\ell 1}$ and $\Phi_{\ell 2}$ parameters to the values given in Table I for the case when the oscillation parameters Δm_{ij}^2 takes the values at BFP. So, the $\chi^2 = \chi^2(\delta_l, \delta_\nu)$ function implies one degree of freedom. In the Fig. 2 we shown

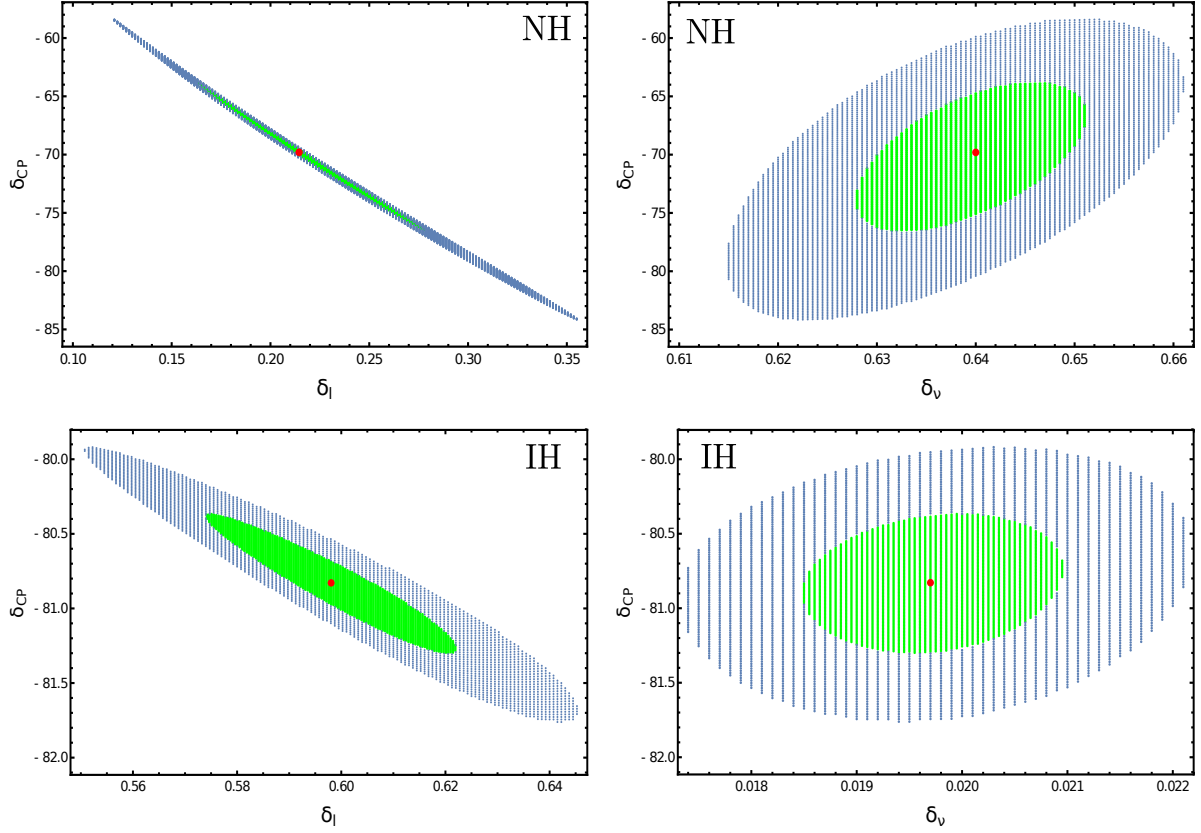


FIG. 3: The allowed regions for “Dirac-like” CP violation phase δ_{CP} and the free parameter δ_l and δ_ν . The $m_{\nu_{1[3]}}$, $\Phi_{\ell 1}$ and $\Phi_{\ell 2}$ parameters are fixing to the values given in Table I for when the oscillation parameters Δm_{ij}^2 takes the values at BFP. The BFP is denoted by red point \bullet , the green and blue regions are for 1σ and 3σ C.L., respectively. The upper and lower panels correspond to normal and inverted hierarchy, respectively.

the allowed regions in the parameter space for δ_l and δ_ν at 3σ C.L., the red point (\bullet) represent to the BFP. The left panel is for normal hierarchy and we can see that δ_ν and δ_l parameters are of the order of 10^{-1} . The right panel is for the inverted hierarchy and we can see that δ_ν parameter is of the order of 10^{-1} , while the δ_l parameter is of the order of 10^{-2} .

Associated to parameter regions of δ_ν and δ_l given in Fig. 2, for both hierachies on neutrino mass spectrum and based on Eq. 33, we found the predicted regions by our model $\nu 2HDM \otimes S_3$ for “Dirac-like” phase δ_{CP} , these regions are shown in Fig. 3. In concordance with experimental data, plots as function of δ_l are more restricted than the plots as function of δ_ν . These results are in very good agreement with allowed regions obtained in the global fit reported in Ref. [15].

In the same way, for both hierarchies we analyze the three leptonic flavor mixing angles, but in the Fig. 4 we just show the allowed regions for the atmospheric mixing angle θ_{23} , at BFP, $\pm 1\sigma$ and 3σ C.L. In order to round the above results, from our analysis we obtain the following values for the three mixing angles, at BFP $\pm 1\sigma$ C.L.:

$$\sin^2 \theta_{12}^{\text{th}}(10^{-1}) = \begin{cases} 3.09^{+0.066}_{-0.065} \\ 3.10^{+0.011}_{-0.011} \end{cases}, \quad \sin^2 \theta_{23}^{\text{th}}(10^{-1}) = \begin{cases} 4.41^{+0.10}_{-0.14} \\ 5.87^{+0.224}_{-0.223} \end{cases}, \quad \sin^2 \theta_{13}^{\text{th}}(10^{-2}) = \begin{cases} 2.160 \pm 0.14 \\ 2.177 \pm 0.12 \end{cases}. \quad (40)$$

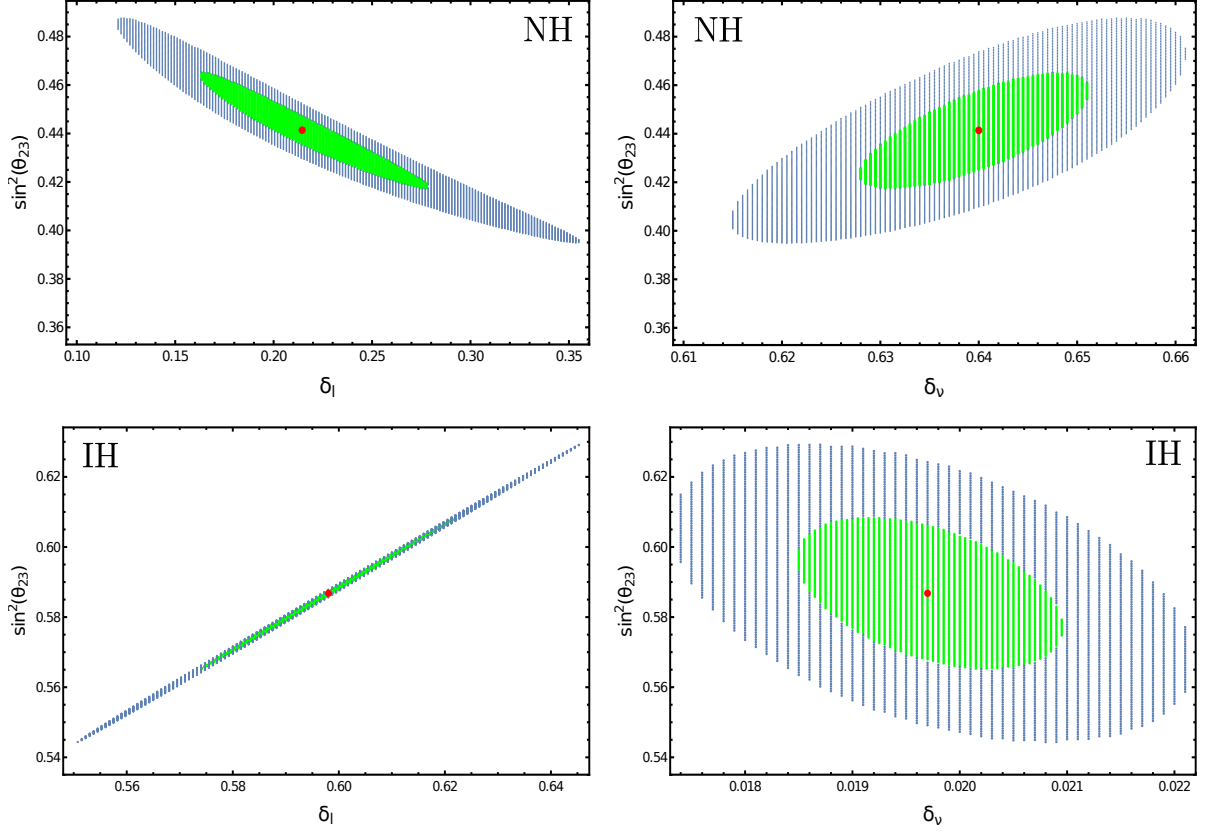


FIG. 4: The allowed regions for atmospheric mixing angle $\sin^2 \theta_{23}$ and the free parameter δ_l and δ_ν . The $m_{\nu_{1[3]}}$, $\Phi_{\ell 1}$ and $\Phi_{\ell 2}$ parameters are fixing to the values given in Table I for when the oscillation parameters Δm_{ij}^2 takes the values at BFP. The BFP is denoted by red point \bullet , the green and blue regions are for 1σ and 3σ C.L., respectively. The upper and lower panels correspond to normal and inverted hierarchy, respectively.

We also obtained the following allowed value ranges at $\text{BFP} \pm 1\sigma$ for the “Dirac-like” phase δ_{CP} , as well as for the two Majorana factors ϕ_{12} and ϕ_{13} :

$$\delta_{CP}(^{\circ}) = \begin{cases} -69.8_{-6.110}^{+5.508} \\ -80.83_{-0.709}^{+0.652} \end{cases}, \quad \phi_{12}(^{\circ}) = \begin{cases} -5.800_{-0.150}^{+0.170} \\ -5.24_{-0.148}^{+0.153} \end{cases}, \quad \phi_{13}(^{\circ}) = \begin{cases} 14.744_{-1.366}^{+1.266} \\ -2.190_{-0.0005}^{+0.0030} \end{cases}. \quad (41)$$

From Eq. (41) and the Table I we can conclude that values for the δ_{CP} phase obtained in our scheme are consistent with a maximal CP violation.

Finally, as a immediate result of the above likelihood analysis, the entries magnitude of \mathbf{V}_{PMNS} mixing matrix can numerically computed. So, at 3σ C.L., we have that \mathbf{V}_{PMNS} matrix takes the form:

$$\begin{pmatrix} 0.822_{-0.0045}^{+0.0044} & 0.550_{-0.0054}^{+0.0055} & 0.147_{-0.0048}^{+0.0047} \\ 0.395_{-0.0154}^{+0.0181} & 0.642_{-0.0001}^{+0.0008} & 0.657_{-0.0111}^{+0.0082} \\ 0.410_{-0.0089}^{+0.0056} & 0.534_{-0.0056}^{+0.0045} & 0.739_{-0.0064}^{+0.0088} \end{pmatrix}, \quad \text{Normal Hierarchy}, \quad (42)$$

$$\begin{pmatrix} 0.822^{+0.0012}_{-0.0012} & 0.551^{+0.0007}_{-0.0006} & 0.147^{+0.0041}_{-0.0041} \\ 0.355^{+0.0072}_{-0.0071} & 0.547^{+0.0144}_{-0.0149} & 0.758^{+0.0138}_{-0.0141} \\ 0.446^{+0.0077}_{-0.0080} & 0.630^{+0.0120}_{-0.0122} & 0.636^{+0.0174}_{-0.0178} \end{pmatrix}, \quad \text{Inverted Hierarchy.} \quad (43)$$

IV. PHENOMENOLOGICAL IMPLICATIONS

In the above section we seen that in our theoretical framework where the S_3 flavor symmetry set that fermion mass matrices should have two texture zeros, we can reproduce the values of oscillation parameters in very good agreement with the last experimental data. In the following, we shall investigate the phenomenological implications of these results for the neutrinoless double beta decay ($0\nu\beta\beta$) and the CP violation in neutrino oscillations in matter.

A. Neutrinoless double beta decay

The $0\nu\beta\beta$ is a rare second-order weak process where a nucleus (A, Z) decays into another one by the emission of two electrons, whose mode decay is $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$. The observation of this process would establish that neutrino are Majorana particles and that total lepton number is not a conserved symmetry in nature [41, 42]. In the most simple version of the process, the amplitude for the decay is proportional to a quantity called the effective mass m_{ee} [43]. In the symmetric parametrization of lepton mixing matrix the effective mass parameter have the shape [44, 45]

$$|m_{ee}| = \left| m_{\nu_1} \cos^2 \theta_{12} \cos^2 \theta_{13} + m_{\nu_2} \sin^2 \theta_{12} \cos^2 \theta_{13} e^{-i2\phi_{12}} + m_{\nu_3} \sin^2 \theta_{13} e^{-i2\phi_{13}} \right|, \quad (44)$$

where ϕ_{12} and ϕ_{13} are the Majorana phases given in Eq. (33). In the Fig. 5 we show the allowed regions for the magnitude of effective mass parameter m_{ee} , which were obtained in the context of $\nu 2\text{HDM} \otimes S_3$. Each one of these regions was obtained by setting the values of some of five free parameters in the χ^2 function, Eq. (37), to the values given in the Table I for Δm_{ij}^2 at BFP. Then, for both hierachies in the lower panels of Fig. 5, the blue lines were obtained by means a likelihood test where the values of $\phi_{\ell 1}$, $\phi_{\ell 2}$, δ_e and δ_ν are fixed, while $m_{\nu_{\text{lightest}}}$ is free parameter. The orange bands were obtained by means a likelihood test where the values of $\phi_{\ell 2}$, δ_e and δ_ν are fixed, while $m_{\nu_{\text{lightest}}}$ and $\phi_{\ell 1}$ are free parameters. The yellow bands were obtained by means a likelihood test where the values of $\phi_{\ell 1}$, δ_e and δ_ν are fixed, while $m_{\nu_{\text{lightest}}}$ and $\phi_{\ell 2}$ are free parameters. The sky blue bands were obtained by means a likelihood test where the values of $\phi_{\ell 1}$, $\phi_{\ell 2}$ and δ_ν are fixed, while $m_{\nu_{\text{lightest}}}$ and δ_e are free parameters. Finally, the turquoise bands were obtained by means a likelihood test where the values of $\phi_{\ell 1}$, $\phi_{\ell 2}$ and δ_e are fixed, while $m_{\nu_{\text{lightest}}}$ and δ_ν are free parameters.

In order to round the previous results, in the table II we show the allowed numerical ranges, at 95% CL, for the magnitude of effective mass parameter m_{ee} and the lightest neutrino mass $m_{\nu_{\text{lightest}}}$. From the results in the table II it is easy conclude that for the normal hierarchy we have $m_{\nu_1} \sim 2 \times 10^{-3}$ eV and $|m_{ee}| \sim 3 \times 10^{-3}$ eV. While for the inverted hierarchy we have $m_{\nu_3} \sim 2 \times 10^{-2}$ eV and $|m_{ee}| \sim 3 \times 10^{-2}$ eV.

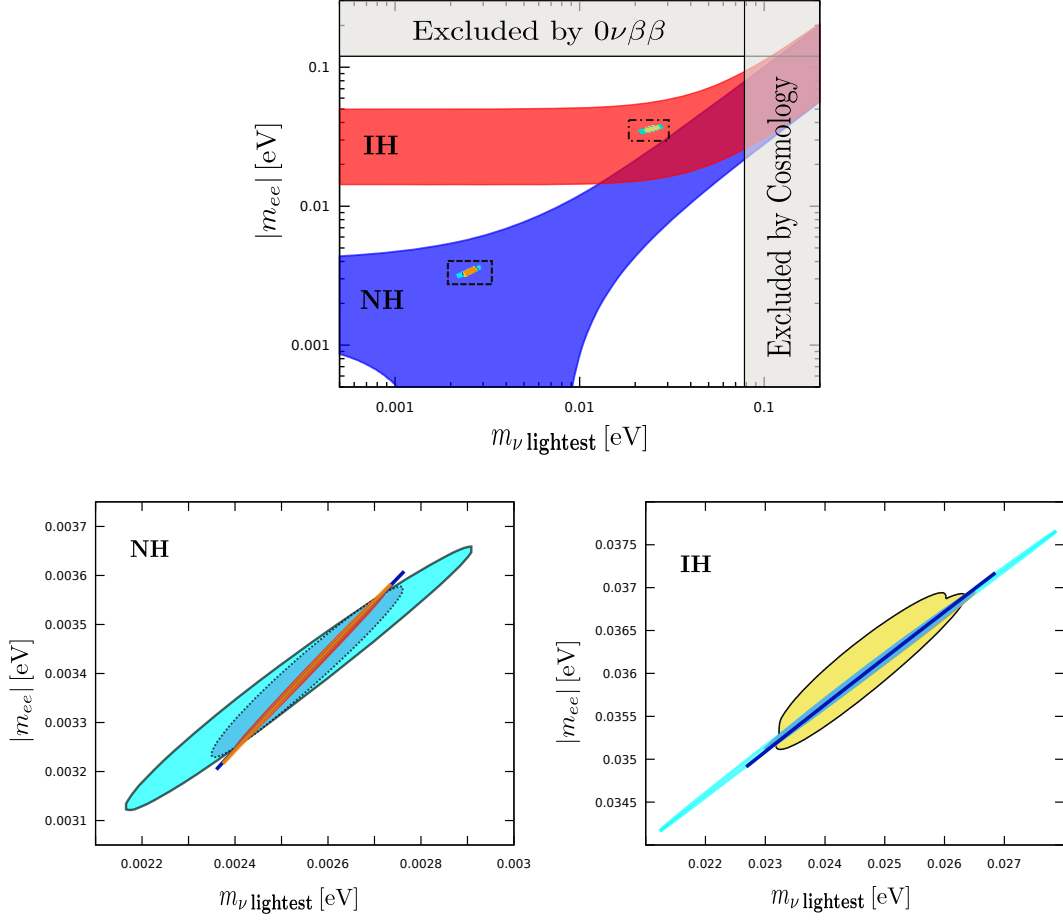


FIG. 5: In the upper panel we show the effective mass $|m_{ee}|$ which is involved in the $0\nu\beta\beta$ decay. The red and blue bands are obtained with the current experimental data on neutrino oscillations, at 3σ [15], for an inverted and normal neutrino mass hierarchy, respectively. On the one hand, from the combination of EXO-200 [38, 39] and KamLAND-ZEN [40] results we have the follow upper bound for $|m_{ee}| < 0.120\text{eV}$. On the other hand, from the results reported by Planck Collaboration we have that $\sum_i m_i < 0.230\text{eV}$ at 95% level [37], thus an upper bound on the lightest neutrino mass is established. In the left- and right-lower panels we show a zoom in of the allowed regions for $|m_{ee}|$ obtained at 95% CL in the context of $\nu 2\text{HDM} \otimes S_3$ for a normal and inverted hierarchy, respectively.

B. CP violation in neutrino oscillations in matter

In the last years we have entered into a precision era in the determination of flavor leptonic mixing angles. However, it is not the same situation for the CP violation in this sector, since has yet to be determined experimentalmente the nuemerial value of CP violation phase. But we have a hunch of where to look; the neutrino oscillations with matter effects [46]. One of the aims of the Long-Baseline neutrino experiments such as T2K [47] and NO ν A [48], as well as the proposed experiment DUNE [49], it is the determination of the “Dirac-like” CP violation phase and other parameters that rule the neutrino oscillations $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$. The transition probability in matter for the oscillation between electron and muon neutrinos, as well as the oscillation between electron and muon antineutrinos,

	Fixed parameters	$m_{\nu_{\text{lightest}}}$ [10^{-2} eV]	$ m_{ee} $ [10^{-2} eV]
NH	$\phi_{\ell 1}, \phi_{\ell 2}, \delta_e, \delta_\nu$	[0.2360 , 0.2768]	[0.3204 , 0.3608]
	$\phi_{\ell 2}, \delta_e, \delta_\nu$	[0.2374 , 0.2735]	[0.3215 , 0.3583]
	$\phi_{\ell 1}, \delta_e, \delta_\nu$	[0.2404 , 0.2711]	[0.3251 , 0.3563]
	$\phi_{\ell 1}, \phi_{\ell 2}, \delta_\nu$	[0.2349 , 0.2761]	[0.3229 , 0.3577]
	$\phi_{\ell 1}, \phi_{\ell 2}, \delta_e$	[0.2164 , 0.2908]	[0.3121 , 0.3659]
IH	$\phi_{\ell 1}, \phi_{\ell 2}, \delta_e, \delta_\nu$	[2.268 , 2.685]	[3.491 , 3.717]
	$\phi_{\ell 2}, \delta_e, \delta_\nu$	[2.317 , 2.635]	[3.511 , 3.694]
	$\phi_{\ell 1}, \delta_e, \delta_\nu$	[2.311 , 2.650]	[3.515 , 3.696]
	$\phi_{\ell 1}, \phi_{\ell 2}, \delta_\nu$	[2.301 , 2.648]	[3.512 , 3.695]
	$\phi_{\ell 1}, \phi_{\ell 2}, \delta_e$	[2.123 , 2.787]	[3.416 , 3.767]

TABLE II: The allowed numerical ranges, at 95% CL, for the magnitude of effective mass parameter m_{ee} and the lightest neutrino mass $m_{\nu_{\text{lightest}}}$.

have the form [45, 50, 51]:

$$\begin{aligned}
P(\nu_\mu \rightarrow \nu_e) &\simeq P_{\text{atm}} + P_{\text{sol}} + 2\sqrt{P_{\text{atm}}}\sqrt{P_{\text{sol}}}\cos(\Delta_{32} + \delta_{\text{CP}}), \\
P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) &\simeq \mathcal{P}_{\text{atm}} + P_{\text{sol}} + 2\sqrt{\mathcal{P}_{\text{atm}}}\sqrt{P_{\text{sol}}}\cos(\Delta_{32} - \delta_{\text{CP}})
\end{aligned} \tag{45}$$

where

$$\begin{aligned}
\sqrt{P_{\text{sol}}} &= \cos\theta_{23}\sin2\theta_{12}\frac{\sin aL}{aL}\Delta_{21}, \\
\sqrt{P_{\text{atm}}} &= \sin\theta_{23}\sin2\theta_{13}\frac{\sin(\Delta_{31}-aL)}{(\Delta_{31}-aL)}\Delta_{31}, \\
\sqrt{\mathcal{P}_{\text{atm}}} &= \sin\theta_{23}\sin2\theta_{13}\frac{\sin(\Delta_{31}+aL)}{(\Delta_{31}+aL)}\Delta_{31}.
\end{aligned} \tag{46}$$

In the above expressions, L is the baseline,

$$\Delta_{ij} = \frac{\Delta m_{ij}^2 L}{4E}, \quad \Delta m_{ij}^2 = m_i^2 - m_j^2 \quad \text{and} \quad a = \frac{G_F N_e}{\sqrt{2}}. \tag{47}$$

Here, E is the energy of neutrino beam, G_F is the Fermi constant and N_e is the density of electrons. The a parameter is $a \approx (3500)^{-1}$ for the earth crust [50]. The asymmetry between $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ in the matter is [51]

$$\begin{aligned}
A_{\mu e} &= \frac{P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}{P(\nu_\mu \rightarrow \nu_e) + P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)} \\
&= \frac{(P_{\text{atm}} - \mathcal{P}_{\text{atm}}) + 2\sqrt{P_{\text{sol}}}(\sqrt{P_{\text{atm}}}\cos(\Delta_{32} + \delta_{\text{CP}}) - \sqrt{\mathcal{P}_{\text{atm}}}\cos(\Delta_{32} - \delta_{\text{CP}}))}{(P_{\text{atm}} + \mathcal{P}_{\text{atm}}) + 2\sqrt{P_{\text{sol}}}(\sqrt{P_{\text{atm}}}\cos(\Delta_{32} + \delta_{\text{CP}}) + \sqrt{\mathcal{P}_{\text{atm}}}\cos(\Delta_{32} - \delta_{\text{CP}})) + 2P_{\text{sol}}}.
\end{aligned} \tag{48}$$

The above asymmetry $A_{\mu e}$ it is basically due to the absence of positrons in the journey of neutrino (anti-neutrino) through the earth. Hence, an neutrino experiment with a long-baseline would be more sensitive to measure this asymmetry.

The T2K neutrino oscillation experiment has a long-baseline of 295 km, while the energy of its neutrino beam has a peak around to 0.6 GeV and width of approximately 0.3 GeV [47]. In the Figs. 6 and 7 we shown the transition probability $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$, as well as the asymmetry $A_{\mu e}$ for T2K experiment.

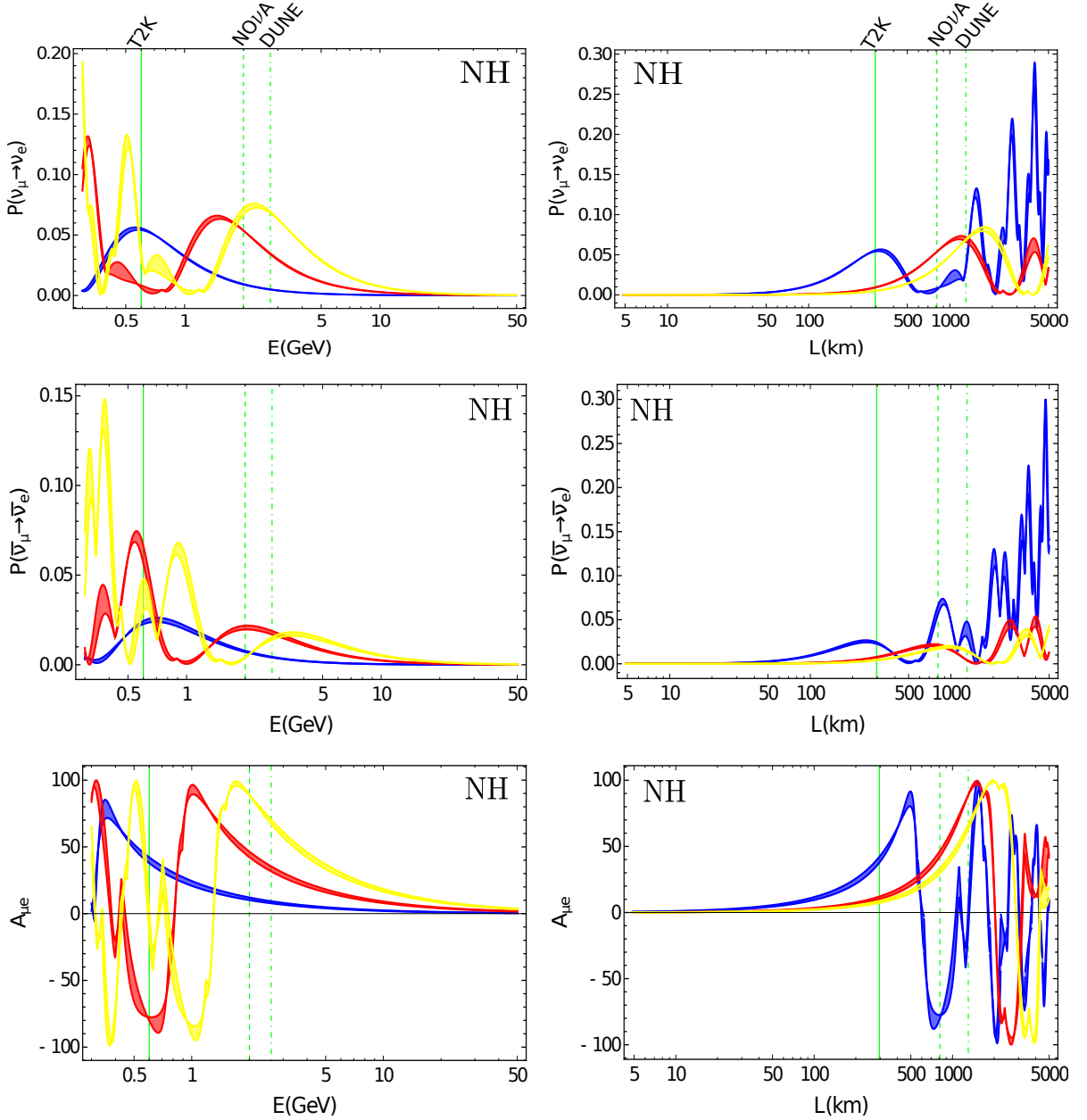


FIG. 6: The $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ transition probabilities, and the asymmetry $A_{\mu e}$ between them, for a normal hierarchy in the neutrino mass spectrum. The blue, red and yellow bands are obtained for a baseline energy L of 295, 810 and 1300 km for left panels, while for the right panels these bands correspond to a neutrino energy E of 0.3, 2 and 2.8 GeV, which correspond to the T2K, NO ν A and DUNE experiment, respectively. Here, the δ_{CP} phase takes values within 1σ C.L. range given in Eq. (41). The remaining parameters are fixed to the values obtained at BFP, which are given in Eq. (34) for Δm_{ij}^2 and Table I for flavor mixing angles.

The NO ν A neutrino oscillation experiment has a long-baseline of 810 km, while the energy of its neutrino beam has a peak around to 2 GeV [48]. In the Fig. 6 and 7 we shown the transition probability $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$, as well as the asymmetry $A_{\mu e}$ for NO ν A experiment.

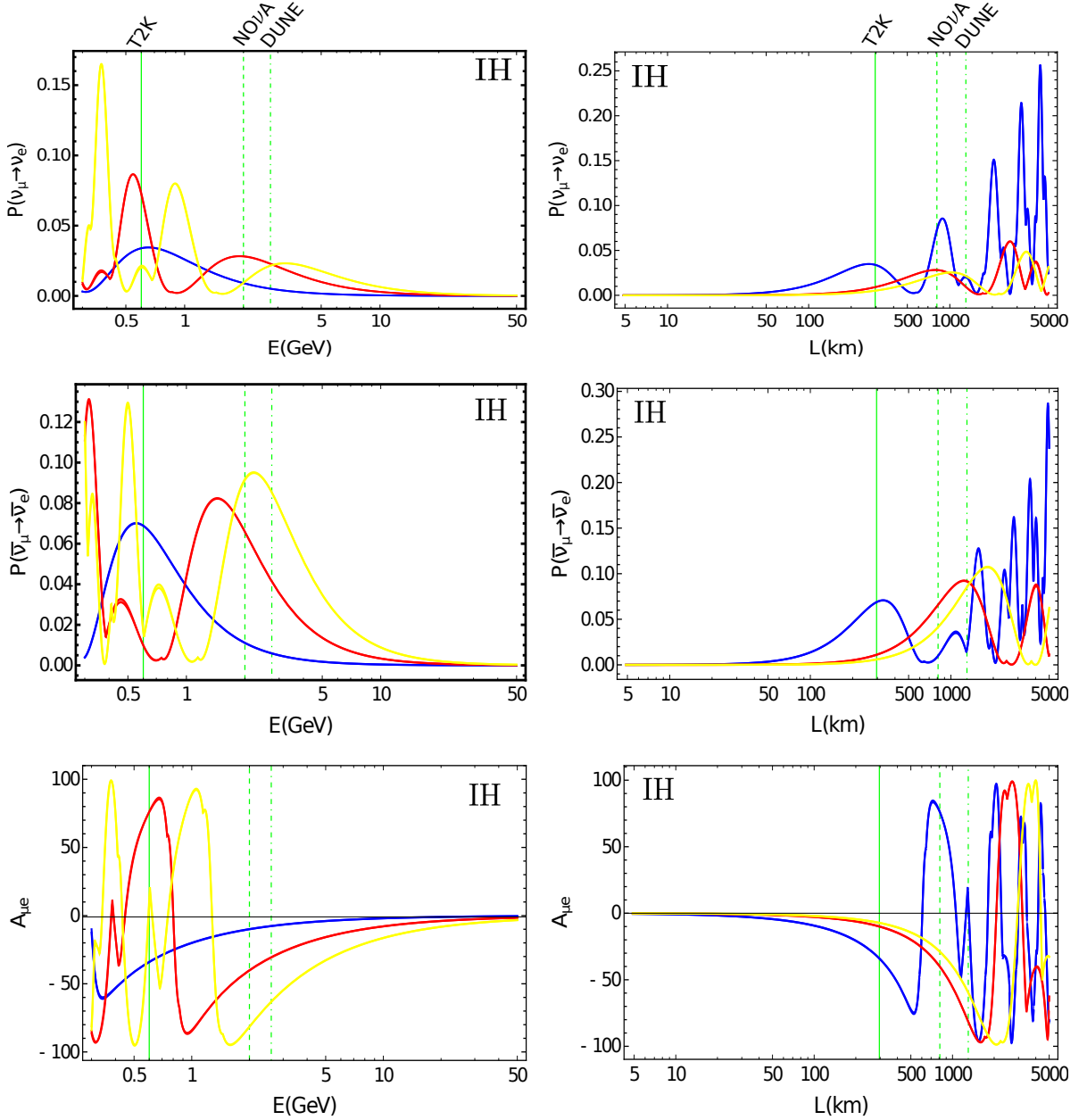


FIG. 7: The $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ transition probabilities, and the asymmetry $A_{\mu e}$ between them, for an inverted hierarchy in the neutrino mass spectrum. The blue, red and yellow bands are obtained for a baseline energy L of 295, 810 and 1300 km for left panels, while for the right panels these bands correspond to a neutrino energy E of 0.3, 2 and 2.8 GeV, which correspond to the T2K, NOνA and DUNE experiment, respectively. Here, the δ_{CP} phase takes values within 1σ C.L. range given in Eq. (41). The remaining parameters are fixed to the values obtained at BFP, which are given in Eq. (34) for Δm_{ij}^2 and Table I for flavor mixing angles.

Finally, the future neutrino oscillation experiment DUNE will have a long baseline of ~ 1300 km, while the energy of its neutrino beam will have a peak around to 2.5 – 3.0 GeV [49]. In the Fig. 6 and 7 we shown the transition probability $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$, as well as the asymmetry $A_{\mu e}$ for DUNE experiment.

V. CONCLUSIONS

We have studied the theory of neutrino masses, mixings and CP violation as the realization of an S_3 flavor symmetry in the framework of the Two Higgs Doublet Model type III. In this $\nu 2\text{HDM} \otimes S_3$ extension of Standard Model, on the one hand the active neutrinos acquire their little masses via the type I seesaw mechanism. On the other hand, the explicit breaking of flavor symmetry according the chain $S_{3L}^j \otimes S_{3R}^j \supset S_3^{\text{diag}} \supset S_2^{\text{diag}}$, allow us to represent in the flavor basis to the Yukawa matrices with an Hermitian matrix with two texture zeros. Consequently, we obtained an unified treatment for all fermionic mass matrices in the model, which are represented through of a matrix with two texture zeros.

The unitary matrices that diagonalize to the mass matrices are expressed in terms of fermion mass ratios. Then, the entries of the Yukawa matrices in the mass basis naturally acquire the form of the so-called *Cheng-Sher ansatz*. Also, the lepton flavor mixing matrix PMNS is expressed as function of the masses of charged leptons and neutrinos, two phases associated with the CP-violation, and two parameters associated with the flavor symmetry breaking. The unitary matrix that allows us to pass from the weak basis to the flavor symmetry adapted basis, is unobservable in the Higgs-fermion-fermion couplings and lepton flavor mixing matrix.

In order to validate our hypothesis where the S_3 horizontal flavor symmetry is explicitly breaking according to the chain $S_{3L}^j \otimes S_{3R}^j \supset S_3^{\text{diag}} \supset S_2^{\text{diag}}$, hence all fermion mass matrices are represented through a matrix with two texture zeros. Furthermore, we make a likelihood test where we compare the theoretical expressions of the flavor mixing angles with the current experimental data on masses and flavor mixing of leptons. The results obtained in this χ^2 analysis are in very good agreement with the current experimental data.

We also obtained the following allowed value ranges, at $\text{BFP} \pm 1\sigma$ for the “Dirac-like” phase factor, as well as for the two Majorana phase factors:

$$\delta_{CP}(^{\circ}) = \begin{Bmatrix} -69.8_{-6.110}^{+5.508} \\ -80.83_{-0.709}^{+0.652} \end{Bmatrix}, \quad \phi_{12}(^{\circ}) = \begin{Bmatrix} -5.800_{-0.150}^{+0.170} \\ -5.24_{-0.148}^{+0.153} \end{Bmatrix}, \quad \phi_{13}(^{\circ}) = \begin{Bmatrix} 14.744_{-1.366}^{+1.266} \\ -2.190_{-0.0005}^{+0.0030} \end{Bmatrix}. \quad (49)$$

The upper (lower) row correspond to the normal (inverted) hierarchy in the neutrino mass spectrum. These values of the phase factors are in agreement with a maximum CP violation in the neutrino oscillation in matter. Finally, we also analyzed the phenomenological implications of the above numerical values of the CP-violation phases on the neutrinoless double beta decay, as well as for long-baseline neutrino oscillation experiments such as T2K, NO ν A, and DUNE.

Appendix A: Three dimensional representation of S_3

The permutations of symmetry group S_3 can be represented on the reducible triplet as [21, 22]:

$$\begin{aligned} \mathbf{D}^{(3)}(E) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{D}^{(3)}(A_1) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{D}^{(3)}(A_2) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \\ \mathbf{D}^{(3)}(A_3) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{D}^{(3)}(A_4) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{D}^{(3)}(A_5) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \end{aligned} \quad (\text{A1})$$

In this representation the projection operators takes the form:

$$\begin{aligned} \text{Symmetric singlet,} \quad \mathbf{P}_1 &= \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = |v_1\rangle\langle v_1|. \\ \text{Anti-symmetric singlet,} \quad \mathbf{P}_{1'} &= 0. \\ \text{Doublet,} \quad \mathbf{P}_2 &= \frac{1}{3} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} = |v_{2A}\rangle\langle v_{2A}| + |v_{2S}\rangle\langle v_{2S}|. \end{aligned} \quad (\text{A2})$$

Here, the vector $|v_1\rangle = \frac{1}{\sqrt{3}}(1, 1, 1)^\top$ is associated with the symmetric singlet. In the projection operator \mathbf{P}_2 , we have the vectors $|v_{2A}\rangle = \frac{1}{\sqrt{2}}(-1, 1, 0)^\top$ and $|v_{2S}\rangle = \frac{1}{\sqrt{2}}(1, 1, -2)^\top$, which are associated with the doublet. Respectively, the vectors $|v_{2A}\rangle$ and $|v_{2S}\rangle$ are antisymmetric and symmetric, under the permutation of first two elements. With the previous three vectors we can construct some tensors that can be helpful. Then,

$$|v_{2S}\rangle\langle v_{2A}| = \frac{1}{\sqrt{12}} \begin{pmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ 2 & -2 & 0 \end{pmatrix} \quad \text{and} \quad |v_{2A}\rangle\langle v_{2S}| = \frac{1}{\sqrt{12}} \begin{pmatrix} -1 & -1 & 2 \\ 1 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}. \quad (\text{A3})$$

If we define the tensors $\mathbf{T}_x^+ = |v_{2S}\rangle\langle v_{2A}| + |v_{2A}\rangle\langle v_{2S}|$ and $\mathbf{T}_x^- = i(|v_{2A}\rangle\langle v_{2S}| - |v_{2S}\rangle\langle v_{2A}|)$ we obtain

$$\mathbf{T}_x^+ = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{T}_x^- = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & i & -i \\ -i & 0 & i \\ i & -i & 0 \end{pmatrix}. \quad (\text{A4})$$

The terms proportional to tensors \mathbf{T}_x^+ and \mathbf{T}_x^- mix the components of the doublet representation each other. Now,

$$|v_1\rangle\langle v_{2A}| = \frac{1}{\sqrt{6}} \begin{pmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 0 \end{pmatrix} \quad \text{and} \quad |v_{2A}\rangle\langle v_1| = \frac{1}{\sqrt{6}} \begin{pmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}. \quad (\text{A5})$$

If we define the tensors $\mathbf{T}_y^+ = |v_1\rangle\langle v_{2A}| + |v_{2A}\rangle\langle v_1|$ and $\mathbf{T}_y^- = i(|v_{2A}\rangle\langle v_{2A}| - |v_{2A}\rangle\langle v_{2S}|)$ we obtain

$$\mathbf{T}_y^+ = \frac{1}{\sqrt{6}} \begin{pmatrix} -2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{T}_y^- = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 & 2i & i \\ -2i & 0 & -i \\ -i & i & 0 \end{pmatrix}. \quad (\text{A6})$$

The terms proportional to tensors \mathbf{T}_y^+ and \mathbf{T}_y^- mix the antisymmetric component of doublet with the singlet. Finally,

$$|v_1\rangle\langle v_{2S}| = \frac{1}{3\sqrt{2}} \begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ 1 & 1 & -2 \end{pmatrix} \quad \text{and} \quad |v_{2S}\rangle\langle v_1| = \frac{1}{3\sqrt{2}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix}. \quad (\text{A7})$$

If we define the tensors $\mathbf{T}_z^+ = |v_1\rangle\langle v_{2S}| + |v_{2S}\rangle\langle v_1|$ and $\mathbf{T}_y^- = i(|v_{2A}\rangle\langle v_{2A}| - |v_{2A}\rangle\langle v_{2S}|)$ we obtain

$$\mathbf{T}_z^+ = \frac{1}{3\sqrt{2}} \begin{pmatrix} 2 & 2 & -1 \\ 2 & 2 & -1 \\ -1 & -1 & -4 \end{pmatrix} \quad \text{and} \quad \mathbf{T}_z^- = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & -i \\ i & i & 0 \end{pmatrix}. \quad (\text{A8})$$

The terms proportional to tensors \mathbf{T}_z^+ and \mathbf{T}_z^- mix the symmetric component of doublet with the singlet. The tensor \mathbf{T}_z^+ can be written as a linear combination of two independent matrices,

$$\mathbf{T}_z^+ = \sqrt{\frac{2}{3}} \mathbf{T}_{z1}^+ - \frac{1}{3\sqrt{2}} \mathbf{T}_{z2}^+, \quad (\text{A9})$$

where

$$\mathbf{T}_{z1}^+ = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad \text{and} \quad \mathbf{T}_z^- = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}. \quad (\text{A10})$$

Appendix B: Cheng-Sher Parameters

$$\begin{aligned} \tilde{\mathbf{Y}}_k^j &= \mathbf{U}_j^\dagger \mathbf{Y}_k^{w,j} \mathbf{U}_j = \mathbf{O}_j^\top \mathbf{P}_j^\dagger \mathbf{U}_S^\dagger \mathbf{Y}_k^{w,j} \mathbf{U}_S \mathbf{P}_j \mathbf{O}_j, \\ \left(\tilde{\mathbf{Y}}_k^j \right)_{rs} &= \frac{\sqrt{m_{j\mathbf{r}} m_{j\mathbf{s}}}}{v} \left(\tilde{\chi}_k^j \right)_{rs} \end{aligned} \quad (\text{B1})$$

where

$$\begin{aligned} \left(\tilde{\chi}_k^j \right)_{11} &= 2 \frac{\xi_{j1}}{D_{j1}} \sqrt{\frac{\hat{m}_{j2}}{\hat{m}_{j1}}} (1 - \delta_j) \cos(\phi_k^j - \phi_j) \tilde{a}_k^j + \frac{(1 - \delta_j) \xi_{j1} \xi_{j2}}{D_{j1}} \tilde{b}_k^j - 2 \frac{\sqrt{\delta_j(1 - \delta_j) \xi_{j1} \xi_{j2}}}{D_{j1}} \tilde{c}_k^j + \frac{\delta_j \xi_{j2}}{D_{j1}} \tilde{d}_k^j, \\ \left(\tilde{\chi}_k^j \right)_{12} &= \sqrt{\frac{(1 - \delta_j) \xi_{j1} \xi_{j2}}{D_{j1} D_{j2}}} \left(\sqrt{\frac{\hat{m}_{j2}}{\hat{m}_{j1}}} e^{i(\phi_k^j - \phi_j)} - \sqrt{\frac{\hat{m}_{j1}}{\hat{m}_{j2}}} e^{-i(\phi_k^j - \phi_j)} \right) \tilde{a}_k^j + (1 - \delta_j) \sqrt{\frac{\xi_{j1} \xi_{j2}}{D_{j1} D_{j2}}} \tilde{b}_k^j \\ &\quad - (\xi_{j1} + \xi_{j2}) \sqrt{\frac{\delta_j(1 - \delta_j)}{D_{j1} D_{j2}}} \tilde{c}_k^j + \delta_j \sqrt{\frac{\xi_{j1} \xi_{j2}}{D_{j1} D_{j2}}} \tilde{d}_k^j, \\ \left(\tilde{\chi}_k^j \right)_{13} &= \sqrt{\frac{\hat{m}_{j2}}{\hat{m}_{j1}}} \frac{(1 - \delta_j) \delta_j \xi_{j1}}{D_{j1} D_{j3}} \left(\hat{m}_{j1} e^{-i(\phi_k^j - \phi_j)} + e^{i(\phi_k^j - \phi_j)} \right) \tilde{a}_k^j + (1 - \delta_j) \sqrt{\frac{\delta_j \xi_{j1}}{D_{j1} D_{j3}}} \tilde{b}_k^j \\ &\quad + (\xi_{j1} - \delta_j) \sqrt{\frac{(1 - \delta_j) \xi_{j2}}{D_{j1} D_{j3}}} \tilde{c}_k^j - \xi_{j2} \sqrt{\frac{\delta_j \xi_{j1}}{D_{j1} D_{j3}}} \tilde{d}_k^j, \\ \left(\tilde{\chi}_k^j \right)_{22} &= -2 \sqrt{\frac{\hat{m}_{j1}}{\hat{m}_{j2}}} (1 - \delta_j) \frac{\xi_{j2}}{D_{j2}} \cos(\phi_k^j - \phi_j) \tilde{a}_k^j + \frac{(1 - \delta_j) \xi_{j2}}{D_{j2}} \tilde{b}_k^j - 2 \frac{\sqrt{\delta_j(1 - \delta_j) \xi_{j1} \xi_{j2}}}{D_{j2}} \tilde{c}_k^j + \frac{\delta_j \xi_{j2}}{D_{j2}} \tilde{d}_k^j, \\ \left(\tilde{\chi}_k^j \right)_{23} &= -\sqrt{\frac{\hat{m}_{j1}}{\hat{m}_{j2}}} \frac{\delta_j(1 - \delta_j) \xi_{j2}}{D_{j2} D_{j3}} \left(e^{i(\phi_k^j - \phi_j)} - \hat{m}_{j2} e^{-i(\phi_k^j - \phi_j)} \right) \tilde{a}_k^j + (1 - \delta_j) \sqrt{\frac{\delta_j \xi_{j2}}{D_{j2} D_{j3}}} \tilde{b}_k^j \\ &\quad + (\xi_{j2} - \delta_j) \sqrt{\frac{(1 - \delta_j) \xi_{j1}}{D_{j2} D_{j3}}} \tilde{c}_k^j - \xi_{j1} \sqrt{\frac{\delta_j \xi_{j2}}{D_{j2} D_{j3}}} \tilde{d}_k^j, \\ \left(\tilde{\chi}_k^j \right)_{33} &= 2 \sqrt{(1 - \delta_j) \hat{m}_{j1} \hat{m}_{j2}} \frac{\delta_j}{D_{j3}} \cos(\phi_k^j - \phi_j) \tilde{a}_k^j + \frac{\delta_j(1 - \delta_j) \tilde{b}_k^j}{D_{j3}} + 2 \frac{\sqrt{\delta_j(1 - \delta_j) \xi_{j1} \xi_{j2}}}{D_{j3}} \tilde{c}_k^j + \frac{d_{jk} \xi_{j1} \xi_{j2}}{D_{j3}} \tilde{d}_k^j, \end{aligned} \quad (\text{B2})$$

with

$$\tilde{a}_k^j = \frac{v}{m_{j3}} \left| A_k^j \right|, \quad \tilde{b}_k^j = \frac{v}{m_{j3}} B_k^j, \quad \tilde{c}_k^j = \frac{v}{m_{j3}} C_k^j, \quad \text{and} \quad \tilde{d}_k^j = \frac{v}{m_{j3}} D_k^j. \quad (\text{B3})$$

Appendix C: Mixing Matrix

The lepton flavor mixing matrix is

$$\mathbf{U}_{\text{PMNS}} = \mathbf{U}_l^\dagger \mathbf{U}_\nu = \mathbf{O}_l^\top \mathbf{P}_l^\dagger \mathbf{U}_S^\dagger \mathbf{U}_S \mathbf{P}_\nu \mathbf{O}_\nu^{n[i]} = \mathbf{O}_l^\top \mathbf{P}^{(\nu-l)} \mathbf{O}_\nu^{n[i]}. \quad (\text{C1})$$

The explicit form of entries of previous matrix are:

$$\begin{aligned} (\mathbf{U}_{\text{PMNS}})_{11} &= \sqrt{\frac{\hat{m}_\mu \hat{m}_{\nu 2[1]} \xi_{l1} \xi_{\nu 1[3]}}{D_{l1} D_{\nu 1[3]}}} + \sqrt{\frac{\hat{m}_e \hat{m}_{\nu 1[3]}}{D_{l1} D_{\nu 1[3]}}} \left(\sqrt{(1-\delta_\nu)(1-\delta_l)} \xi_{l1} \xi_{\nu 1[3]} e^{i\phi_{l1}} + \sqrt{\delta_\nu \delta_l \xi_{l2} \xi_{\nu 2[1]}} e^{i\phi_{l2}} \right), \\ (\mathbf{U}_{\text{PMNS}})_{12} &= -\sqrt{\frac{\hat{m}_\mu \hat{m}_{\nu 1[3]} \xi_{l1} \xi_{\nu 2[1]}}{D_{l1} D_{\nu 2[1]}}} + \sqrt{\frac{\hat{m}_e \hat{m}_{\nu 2[1]}}{D_{l1} D_{\nu 2[1]}}} \left(\sqrt{(1-\delta_\nu)(1-\delta_l)} \xi_{l1} \xi_{\nu 2[1]} e^{i\phi_{l1}} + \sqrt{\delta_\nu \delta_l \xi_{l2} \xi_{\nu 1[3]}} e^{i\phi_{l2}} \right), \\ (\mathbf{U}_{\text{PMNS}})_{13} &= \sqrt{\frac{\hat{m}_\mu \hat{m}_{\nu 1[3]} \hat{m}_{\nu 2[1]} \delta_\nu \xi_{l1}}{D_{l1} D_{\nu 3[2]}}} + \sqrt{\frac{\hat{m}_e}{D_{l1} D_{\nu 3[2]}}} \left(\sqrt{(1-\delta_\nu) \delta_\nu (1-\delta_l)} \xi_{l1} e^{i\phi_{l1}} - \sqrt{\delta_l \xi_{l2} \xi_{\nu 1[3]} \xi_{\nu 2[1]}} e^{i\phi_{l2}} \right), \\ (\mathbf{U}_{\text{PMNS}})_{21} &= -\sqrt{\frac{\hat{m}_e \hat{m}_{\nu 2[1]} \xi_{l2} \xi_{\nu 1[3]}}{D_{l2} D_{\nu 1[3]}}} + \sqrt{\frac{\hat{m}_\mu \hat{m}_{\nu 1[3]}}{D_{l2} D_{\nu 1[3]}}} \left(\sqrt{(1-\delta_\nu)(1-\delta_l)} \xi_{l2} \xi_{\nu 1[3]} e^{i\phi_{l1}} + \sqrt{\delta_\nu \delta_l \xi_{l1} \xi_{\nu 2[1]}} e^{i\phi_{l2}} \right), \\ (\mathbf{U}_{\text{PMNS}})_{22} &= \sqrt{\frac{\hat{m}_e \hat{m}_{\nu 1[3]} \xi_{l2} \xi_{\nu 2[1]}}{D_{l2} D_{\nu 2[1]}}} + \sqrt{\frac{\hat{m}_\mu \hat{m}_{\nu 2[1]}}{D_{l2} D_{\nu 2[1]}}} \left(\sqrt{(1-\delta_\nu)(1-\delta_l)} \xi_{l2} \xi_{\nu 2[1]} e^{i\phi_{l1}} + \sqrt{\delta_\nu \delta_l \xi_{l1} \xi_{\nu 1[3]}} e^{i\phi_{l2}} \right), \\ (\mathbf{U}_{\text{PMNS}})_{23} &= -\sqrt{\frac{\hat{m}_e \hat{m}_{\nu 1[3]} \hat{m}_{\nu 2[1]} \delta_\nu \xi_{l2}}{D_{l2} D_{\nu 3[2]}}} + \sqrt{\frac{\hat{m}_\mu}{D_{l2} D_{\nu 3[2]}}} \left(\sqrt{\delta_\nu (1-\delta_\nu)(1-\delta_l)} \xi_{l2} e^{i\phi_{l1}} - \sqrt{\delta_l \xi_{l1} \xi_{\nu 1[3]} \xi_{\nu 2[1]}} e^{i\phi_{l2}} \right), \\ (\mathbf{U}_{\text{PMNS}})_{31} &= \sqrt{\frac{\hat{m}_e \hat{m}_\mu \hat{m}_{\nu 2[1]} \delta_l \xi_{\nu 1[3]}}{D_{l3} D_{\nu 1[3]}}} + \sqrt{\frac{\hat{m}_{\nu 1[3]}}{D_{l3} D_{\nu 1[3]}}} \left(\sqrt{\delta_l (1-\delta_\nu)(1-\delta_l)} \xi_{\nu 1[3]} e^{i\phi_{l1}} - \sqrt{\delta_\nu \xi_{l1} \xi_{l2} \xi_{\nu 2[1]}} e^{i\phi_{l2}} \right), \\ (\mathbf{U}_{\text{PMNS}})_{32} &= -\sqrt{\frac{\hat{m}_e \hat{m}_\mu \hat{m}_{\nu 1[3]} \delta_l \xi_{\nu 2[1]}}{D_{l3} D_{\nu 2[1]}}} + \sqrt{\frac{\hat{m}_{\nu 2[1]}}{D_{l3} D_{\nu 2[1]}}} \left(\sqrt{\delta_l (1-\delta_\nu)(1-\delta_l)} \xi_{\nu 2[1]} e^{i\phi_{l1}} - \sqrt{\delta_\nu \xi_{l1} \xi_{l2} \xi_{\nu 1[3]}} e^{i\phi_{l2}} \right), \\ (\mathbf{U}_{\text{PMNS}})_{33} &= \sqrt{\frac{\hat{m}_e \hat{m}_\mu \hat{m}_{\nu 1[3]} \hat{m}_{\nu 2[1]} \delta_l \delta_\nu}{D_{l3} D_{\nu 3[2]}}} + \sqrt{\frac{1}{D_{l3} D_{\nu 3[2]}}} \left(\sqrt{\delta_l \delta_\nu (1-\delta_\nu)(1-\delta_l)} e^{i\phi_{l1}} - \sqrt{\xi_{l1} \xi_{l2} \xi_{\nu 1[3]} \xi_{\nu 2[1]}} e^{i\phi_{l2}} \right). \end{aligned} \quad (\text{C2})$$

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